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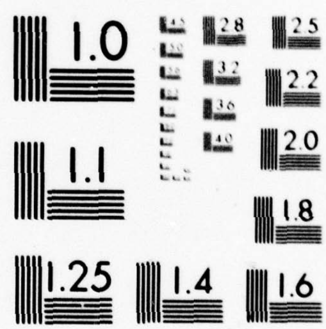

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A COVARIANCE-APPROXIMATION PROCEDURE

Hans [illegible]

The Ohio State University
Research Foundation
Columbus, Ohio 43212

March 1979

Scientific Report No. 1

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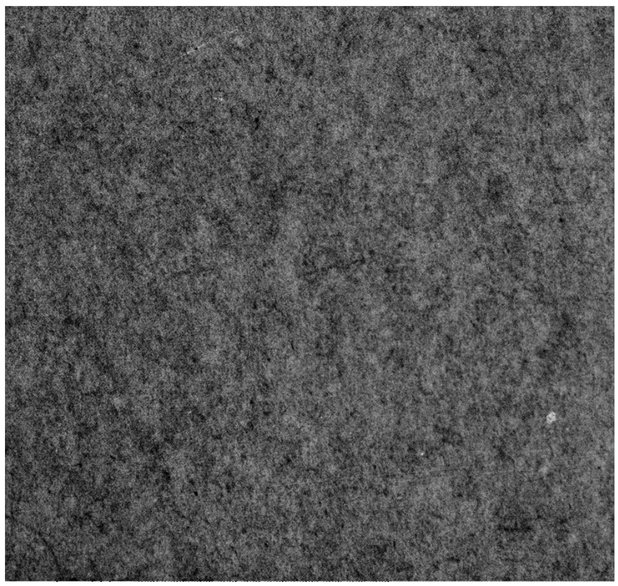

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The steadily increasing stream of geodetic data primarily provided by automatized measurement techniques calls for a data processing method which is both simple and fast. The simplicity of least-squares collocation can hardly be surpassed, however, the processing speed, which depends on covariance calculation procedure and matrix inversion algorithms used, can be improved considerably by the faster covariance computation procedure presented in this report.

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The method is basically a differentiation - interpolation procedure applied to a smooth function interpolating on a regular rectangular grid of exact covariances. Once this covariance network has been generated, the covariance approximation procedure, called COVAPP, is able to provide covariance expressions of second and lower order derivatives of the disturbing potential about ten times faster than existing methods. The approximation error can be kept as small as we please by choosing an appropriately small grid spacing. A FORTRAN IV program listing together with sample inputs and outputs are included in the report.

The reader only interested in application may bypass the technical chapters 3 and 4.

FOREWORD

This report was prepared by Dr. Hans Sünkel, assistant to Dr. Helmut Moritz, Professor, Technical University at Graz, Austria and Adjunct Professor, Department of Geodetic Science of The Ohio State University, under Air Force Contract F19628-79-C-0075, The Ohio State University Research Foundation, Project 711715, Project Supervisor, Urho A. Uotila, Professor, Department of Geodetic Science. The contract covering this research is administered by the Air Force Geophysics Laboratory, Hanscom Air Force Base, Massachusetts, with Mr. Bela Szabo, Contract Manager.

CONTENTS

1. Introduction -- 1

2. The Spatial Covariance Function and its Approximation Model -- 2

3. Covariance Grid Point Values -- 3

4. Covariance Expressions -- 11

5. Program Guideline -- 18

6. Conclusions -- 22

7. References -- 23

Appendix A: A Fortran IV Covariance Approximation Procedure ----- 24

Appendix B: Sample Inputs and Outputs -- 44

1. Introduction

The era of extracting gravity field information about the earth from only time-consuming laborious surface measurements is definitely over. Modern technology has created highly sophisticated instruments which enable us to obtain different kinds of geodetic data with increasing accuracy at a speed never known before. Probably the only technique capable of processing all these data in a consistent way is the method of least-squares collocation, which is as beautiful as deep (Moritz, 1972).

The central role in collocation is played by the model covariance function which resembles somehow the main features of the earth's gravitational field. The property of the kernel function forces it to be harmonic outside some internal sphere; consequently, its eigen-functions are the spherical harmonics. Homogeneity and isotropy postulates make it dependent on only two essential variables: the spherical distance between two points and the product of the corresponding geocentric radii of these points.

All geodetic data are functionals of the disturbing potential. Therefore, all covariances between geodetically relevant quantities can be derived by means of the covariance propagation law. This, again, is the reason why closed analytical expressions of the covariance function are an absolute necessity. Tscherning and Rapp (1974) have derived closed expressions on the basis of different degree variance models and give covariance expressions for geoidal height, gravity anomaly and deflection of the vertical, most commonly used in geodetic applications. Tscherning (1976) extended this work and derived covariance expressions for second order derivatives of the anomalous potential. His very elegant and convenient subroutine COVAX has been widely applied by the geodetic community.

However, the number of problems for which the application of COVAX becomes expensive increases steadily. All these expensive problems

involve the numerical integration of the covariance function resulting in a large number of covariance computations. Two typical examples: a) the prediction of mean gravity anomalies from different kinds of data (Rapp, 1977, 1978), b) satellite-to-staellite tracking especially in the low-low mode) (Kryn'ski, 1978). In both cases multifold integrations of the covariance function are involved (in the first case integrations over "rectangular" areas on the sphere, in the second case integrations along the flight paths of two satellites).

For this reason the possibility had been studied to cut down the considerably the covariance computation time by using approximations of the covariance function (Sünkel, 1978 a,b). Theoretical investigations indicated that this possibility exists; preliminary practical calculations encouraged further detailed studies. After all, a way has been found through the jungle which permitted a consistent approximation of all covariance expressions up to and including second-order derivatives of the disturbing potential. Basically, the method consists of choosing a regular rectangular grid with respect to the cosine of the spherical distance $t_j = \cos\psi_j$ and with respect to the squared ratio s between the radius of the Bjerhammar sphere R_B and some radius $r > R_B$, $s_i = (R_B / r_i)^2$, and storing all covariances corresponding to these grid points on some file. All desired covariance expressions can be derived by a simple differentiation-interpolation procedure. The accuracy of the so obtained covariances can be made arbitrary high by selecting an appropriate small grid spacing.

This procedure provides the user with the same covariances as conveniently as COVAX, but about 10 times faster. The price we pay is mass storage. However, computer development statistics show clearly that the amount of available mass storage increases much more than the speed-up of calculation time. Moreover, since large scale applications

of collocation (for which the procedure described here is designed) are by their own nature restricted to large computer systems with a large central core capacity available, the need for storage can no longer be considered an essential drawback. It's believed that this approximation procedure will be especially valuable for collocation problems which involve the integration of the covariance function.

2. The Spatial Covariance Function and its Approximation Model

Although the choice of a particular covariance function model is of no concern to the approximation procedure (as long as it is isotropic), it is nevertheless necessary to give a short description of its essential properties in order to have a solid starting basis.

The general form of a homogeneous and isotropic covariance function can be expressed by

$$K(P,Q) = \sum_{n=N_0}^{\infty} k_n \left(\frac{R_B^2}{rr'}\right)^{n+1} P_n(\cos\psi_{PQ}), \tag{2-1}$$

where

$P, Q \ldots$	points in space,
$r, r' \ldots$	geocentric radii of P and Q,
$\psi_{PQ} \ldots$	spherical distance between P and Q,
$k_n \ldots$	positive coefficients,
$P_n(\cos\psi)$	Legendre polynomial of degree n,
$N_0 \ldots$	starting value of the summation ($N_0 \geq 2$),
$R_B \ldots$	radius of the Bjerhammar sphere.

$K(P,Q)$ is symmetric with respect to P and Q, expressed by the product $r\,r'$ and the dependence on $\cos\psi_{PQ}$.

$$K(P,Q) = K(Q,P);$$

it is harmonic outside the Bjerhammar sphere $r = R_B$,

$$\Delta_P K(P,Q) = \Delta_Q K(P,Q) = 0.$$

An important property which allows a two-dimensional approximation is its dependence on essentially two variables only, the spherical distance ψ_{PQ} and the product $r r'$. Since $\cos\psi$ can vary only between -1 and +1, and R_B^2/rr' has a minimum value of 0 for $r \to \infty$ and a maximum of 1 for $r = r' = R_B$, the covariance functions domain is the rectangle

$$[-1 \le t \le 1, \ 0 < s < 1] \tag{2-2a}$$

with the variables

$$t := \cos\psi, \quad s := \frac{R_B^2}{rr'}. \tag{2-2b}$$

Operations on or close to the surface of the earth together with locally to regionally restricted use, however, reduce the domain of practical applications considerably. E.g.: Working within a spherical distance range of $0 \le \psi \le 10°$ and within an altitude range from 0 (earth's surface) to 300 km (satellite altitude), the operational domain reduces to

$$[0.985 \le t \le 1, 0.999 \le s \le 0.912].$$

The following figure may illustrate the mapping given by (2-2b):

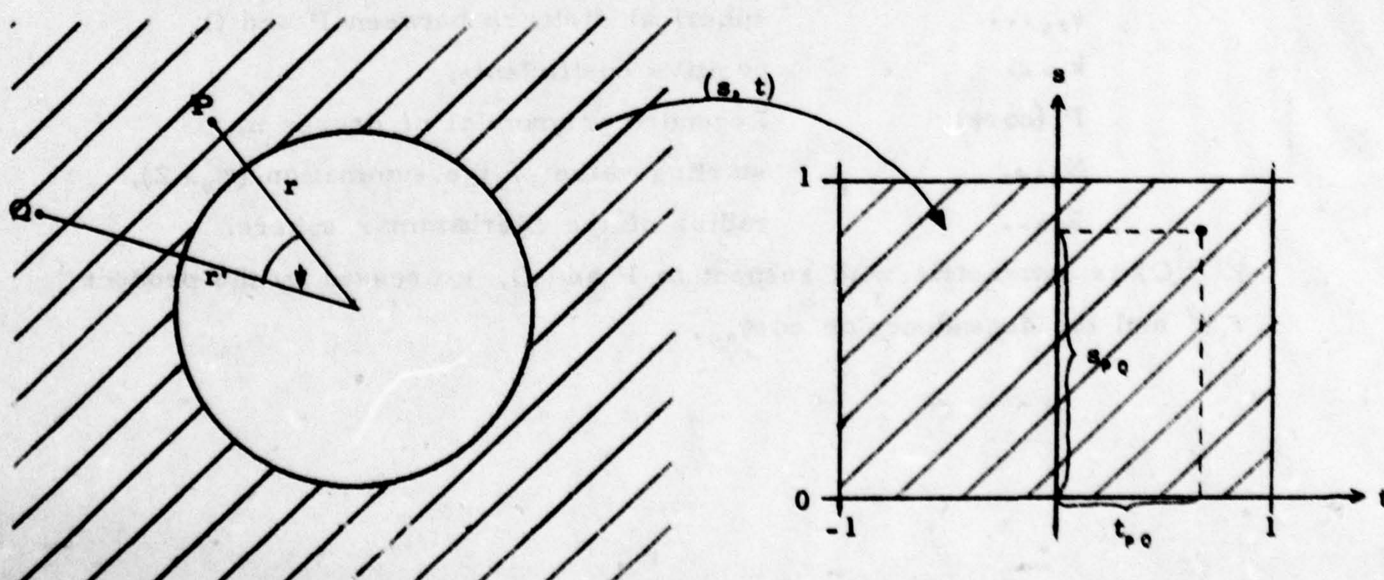

Fig. 1 (s, t) - mapping

Once the operational domain has been fixed, a regular rectangular grid in s and t can be arranged. Two important facts have to be taken into account: The numerical values of the covariances change more rapidly for small ψ than for larger ones, and the covariances become smoother for higher altitudes because of the upward continuation effect. These two statements might sound trivial but they are important, and represent guidelines for the choice of grid distances Δs and Δt. The reason is the following: Approximate covariances are obtained by a differentiation - interpolation procedure applied to some interpolating function uniquely defined by the covariances at the grid points. If the interpolating function is an element of $K_\ell [D]$, the space of continuous functions with domain D, having quadratically integrable first derivatives, it can be shown that the maximum approximation error depends on the $\ell + 1$)th power of the grid spacing multiplied by the maximum absolute value of the ($\ell + 1$)th order derivative of the original function. Each differentiation, if it exists, decreases the dependence on the grid spacing by one power (Sünkel, 1978a, p. 22), (one-dimensional case). Consequently, the grid has to be denser for small spherical distances ψ than for larger ones and denser for small altitudes than for higher ones.

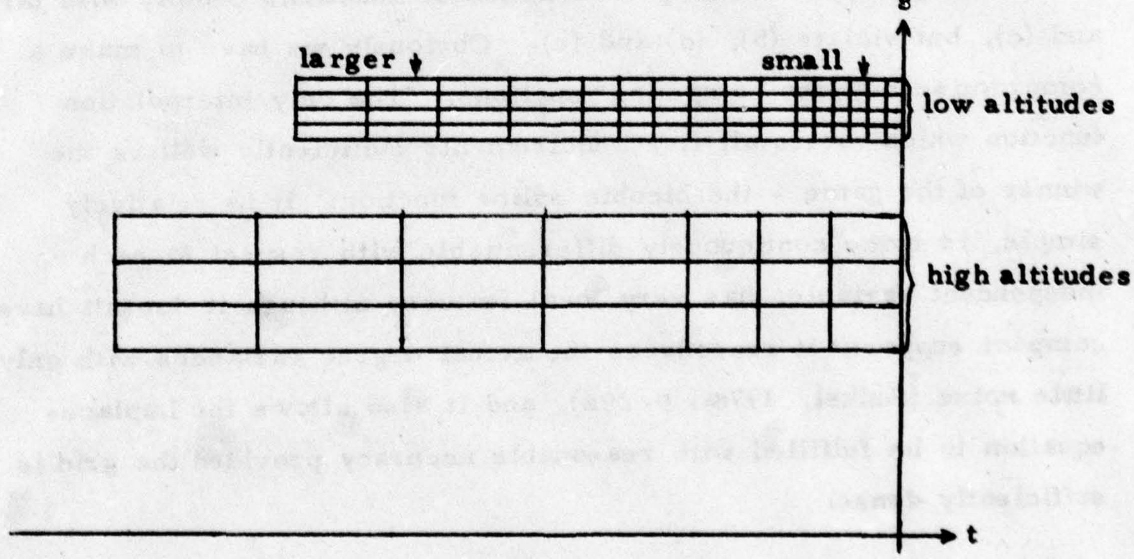

Fig. 2 Grid arrangement principle

Next we have to face the problem of which kind of interpolation function we should choose. Five requirements have to be fulfilled by the interpolation function:

a) It should be as simple as possible in order to keep the computation time at a minimum.

b) It should be as smooth as possible and admit as many continous derivatives as possible.

c) It should be as local as possible in order to guarantee a good fit to the true covariance function not only for large, but also for small ψ -values.

d) It should disturb the true spectrum and the actual degree variances, as little as possible.

e) It should satisfy the Laplace-equation with reasonable accuracy in order to guarantee harmonicity.

Needless to say, these five requirements exclude each other: analytic expressions meet the requirements (b)-(e), but fail at (a); single higher degree polynomials fulfill partly (a) and (b), but violate (c)-(e); step functions and piecewise linear functions comply with (a) and (c), but violate (b), (d) and (e). Obviously we have to make a compromise between competing functions. The only interpolation function which meets all five requirements sufficiently well is the winner of the game - the bicubic spline function: it is relatively simple, is twice continuously differentiable with respect to each independent variable, has very local features although it doesn't have a compact support; it reproduces the actual degree variances with only little noise (Sünkel, 1978a, p. 29a), and it also allows the Laplace-equation to be fulfilled with reasonable accuracy provided the grid is sufficiently dense.

If we were interested in covariances involving not more than three differentiations of the covariance function with respect to s and t, a single bicubic spline representation of the disturbing potential's spatial covariance function would be sufficient.(*) However, we intend to derive approximations of covariance expressions up to second order derivatives (at both points P and Q) and this corresponds to fourth order derivatives with respect to s or t. Consequently, a single spline representation of cov (T, T) is not sufficient anymore, and we need at least two more representations. We have chosen cov (D_r T, $D_{r'}$ T) and cov (D_t T, D_t T) = cov (D_t^2 T, T) in order to guarantee at least linear (and at most cubic) interpolation within the grid. These three spline representations are necessary and sufficient for our purposes. Since the gravity anomaly is the most frequently used quantity in physical geodesy, we have extended the covariance representations and included also the covariance representation of the gravity anomaly covariance function. It should, however, be pointed out that this is not necessary and has been done only for reasons of convenience.

The four covariance functions, cov (T, T), cov (D_r T, $D_{r'}$ T), cov (D_t^2 T, T) and cov (Δg, Δg) depend only on s and t and so do their bicubic spline representations. All other covariance expressions up to (at least) second order derivatives of the disturbing potential can be obtained from linear operations applied to these four covariance representations. This is the very philosophy behind covariance approximations which helps us to reduce drastically the CPU-time.

(*)... This follows from the fact that the 3rd derivative of a cubic spline is a step function.

3. Covariance Grid Point Values

Since a bicubic spline representation of the covariance function will be used, it is necessary to determine all derivatives of the covariance function which, together with their function values at all grid points, make the spline representation unique. Let $D(s, t)$ be a rectangular domain divided into $I \cdot J$ subrectangles by the grid points

$$s_0 \geq s_i \geq s_I, \quad t_0 \geq t_j \geq t_J$$

with (s_i, t_j) grid point coordinates. Then a bicubic spline on D is uniquely defined by (Sünkel, 1978a, p. 74)

a) f_{ij}, $i = 0,\dots,I$; $j = 0,\dots,J$ ...function values at the grid points,

b) $D_s f_{ij}$, $i = 0, I$; $j = 0,\dots,J$ ...first derivatives of the function f with respect to s,

c) $D_t f_{ij}$, $i = 0,\dots,I$; $j = 0, J$ ...first derivatives of the function f with respect to t,

d) $D_{st} f_{ij}$, $i = 0, I$; $j = 0, J$ ...second derivatives of the function f with respect to st.

Here, f stands for one of the four covariance functions $\mathrm{cov}(T, T)$, $\mathrm{cov}(D_r T, D_r T)$, $\mathrm{cov}(D_t^2 T, T)$, $\mathrm{cov}(\Delta g, \Delta g)$. What we need are expressions for first order partial derivatives of all 4 covariances with respect to their independent variables s, t. Since the subroutine COVAX of C.C. Tscherning can provide us with values for these quantities implicitly, we intend to express the derivatives in terms of COVAX covariance outputs. In the next sections we derive these formulas.

3.1 Derivatives of cov(T, T) with respect to s and t

We recall the definition (2-1) of the disturbing potential covariance function which we write in a simplified notation as

$$\mathrm{cov}(T, T) = \sum k_n \, s^{n+1} P_n(t),$$

the partial derivative with respect to s is trivial:

$$D_s \operatorname{cov}(T, T) = \Sigma k_n (n+1) s^n P_n(t).$$

This expression can be easily related to $\operatorname{cov}(-\frac{1}{r} D_r T, T)$ by considering

$$\operatorname{cov}(D_r T, T) = -\frac{1}{r} \Sigma k_n (n+1) s^{n+1} P_n(t),$$

and we obtain

$$D_s \operatorname{cov}(T, T) = \frac{r^2}{s} \operatorname{cov}\left(-\frac{1}{r} D_r T, T\right). \tag{3-1}$$

Even simpler is the calculation of $D_t \operatorname{cov}(T, T)$:

$$D_t \operatorname{cov}(T, T) = \Sigma k_n\, s^{n+1} D_t P_n(t). \tag{3-2}$$

(All closed expressions which we don't write down explicitly here, can be found in Tscherning (1976). Their calculations are part of the subroutine COVAX.)

Similarly we obtain

$$D_s D_t \operatorname{cov}(T, T) = \frac{r^2}{s} \operatorname{cov}\left(-\frac{1}{r} D_r T, D_t T\right). \tag{3-3}$$

3.2 <u>Derivatives of cov (D_r T, $D_{r'}$ T) with respect to s and t</u>

By straightforward derivation we find

$$\operatorname{cov}(D_r T, D_{r'} T) = \frac{1}{rr'} \Sigma k_n (n+1)^2 s^{n+1} P_n(t).$$

Recalling the definition of s, we can also write

$$\operatorname{cov}(D_r T, D_{r'} T) = \frac{s}{R_B^2} \Sigma k_n (n+1)^2 s^{n+1} P_n(t).$$

Applying the differentiation operator D_s we obtain

$$D_s \operatorname{cov}(D_r T, D_{r'} T) = \frac{1}{R_B^2} \Sigma k_n (n+1)^2 (n+2) s^{n+1} P_n(t),$$

which can easily be shown to be equivalent to

$$D_s \operatorname{cov}(D_r T, D_{r'} T) = \left(\frac{R_B}{s}\right)^2 \operatorname{cov}\left(-\frac{1}{r} D_r T, D_{r'}^2 T\right). \tag{3-4}$$

The differentiation operator D_t applied to $\text{cov}(D_r T, D_{r'} T)$ yields

$$D_t \,\text{cov}(D_r T, D_{r'} T) = \frac{s}{R_e^2} \sum k_n \,(n+1)^2 \, s^{n+1} D_t P_n(t) \tag{3-5}$$

and similarly we obtain applying D_t to equ. (3-4)

$$D_s D_t \,\text{cov}(D_r T, D_{r'} T) = \left(\frac{R_e}{s}\right)^2 \text{cov}\left(-\frac{1}{r} D_r D_t T, D_{r'}^2 T\right). \tag{3-6}$$

3.3 <u>Derivatives of $\text{cov}(D_t^2 T, T)$ with respect to s and t</u>

All covariance derivatives of this type are closely related to equation (3-1); we obtain

$$\text{cov}(D_t^2 T, T) = \sum k_n s^{n+1} D_t^2 P_n(t), \tag{3-7}$$

$$D_s \,\text{cov}(D_t^2 T, T) = \frac{r^2}{s} \text{cov}(-\frac{1}{r} D_r T, D_t^2 T), \tag{3-8}$$

$$D_t \,\text{cov}(D_t^2 T, T) = \sum k_n s^{n+1} D_t^3 P_n(t), \tag{3-9}$$

$$D_s D_t \,\text{cov}(D_t^2 T, T) = \frac{r^2}{s} \text{cov}(-\frac{1}{r} D_r D_t T, D_t^2 T). \tag{3-10}$$

3.4 <u>Derivatives of $\text{cov}(\Delta g, \Delta g)$ with respect to s and t</u>

All covariance derivatives of this type can be written down immediately replacing T by Δg in section 3.1:

$$D_s \text{cov}(\Delta g, \Delta g) = -\frac{r}{s} \text{cov}(D_r \Delta g, \Delta g), \tag{3-11}$$

$$D_t \,\text{cov}(\Delta g, \Delta g) = \text{cov}(\Delta g, D_t \Delta g), \tag{3-12}$$

$$D_s D_t \,\text{cov}(\Delta g, \Delta g) = -\frac{r}{s} \text{cov}(D_r \Delta g, D_t \Delta g). \tag{3-13}$$

All covariances given above can be calculated by a simplified version of COVAX. The modifications are described in the listing of the program COVNET.

4. Covariance Expressions

All covariance expressions which can be derived from the bicubic spline approximation model are linear combinations of the grid point covariances listed in chapter 3. For example, if the covariance $\mathrm{cov}(T_P, T_Q)$ is desired, it is necessary to calculate the corresponding s and t values and find the element indices (i, j) such that $s_i \geq s \geq s_{i+1}$ and $t_j \geq t \geq t_{j+1}$. On each rectangle $[s_i, s_{i+1}; t_j, t_{j+1}]$ the spatial covariance function is approximated by a bicubic polynomial with 16 coefficients being linearly related to $\mathrm{cov}(T, T)$, $D_s \mathrm{cov}(T, T)$, $D_t \mathrm{cov}(T, T)$ and $D_s D_t \mathrm{cov}(T, T)$ at the four corners of the corresponding rectangle. To find $\mathrm{cov}(T, T)$ for s and t values not coinciding with one of the corner values, is a simple problem of interpolation. (To find $\mathrm{cov}(T, D_t T)$ for the same point (s, t) would be a differentiation and interpolation problem.)

The general form of the bicubic polynomial defined on the rectangular domain $[s_i, s_{i+1}; t_j, t_{j+1}]$ is

$$f^{(ij)}(s, t) = \sum_{k=0}^{3} \sum_{\ell=0}^{3} a_{k\ell}^{(ij)} (s - s_i)^k (t - t_j)^\ell . \tag{4-1}$$

The 16 coefficients result from linear transformations of f, $D_s f$, $D_t f$ and $D_s D_t f$ taken at the 4 corners of the rectangle:

$$A^{(ij)} = \left\{ a_{k\ell}^{(ij)} \right\} = G^T F H$$

with

$$G = \begin{bmatrix} I & G_1 \\ 0 & G_2 \end{bmatrix}, \quad G_1 = \begin{bmatrix} -3/g^2 & 2/g^3 \\ -2/g & 1/g^2 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 3/g^2 & -2/g^3 \\ -1/g & 1/g^2 \end{bmatrix},$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad g := s_{i+1} - s_i \; ; \tag{4-2}$$

$$H = G \big| g = t_{j+1} - t_j \; ;$$

$$F = \begin{bmatrix} F_{ij} & F_{i,j+1} \\ F_{i+1,j} & F_{i+1,j+1} \end{bmatrix}, \quad F_{ij} = \begin{bmatrix} f_{ij} & D_t f_{ij} \\ D_s f_{ij} & D_s D_t f_{ij} \end{bmatrix}.$$

Simple matrix multiplications yield

$$A = \left[\begin{array}{c|c} F_1 & F_1H_1 + F_3H_2 \\ \hline G_1^T F_1 + G_2^T F_2 & (G_1^T F_1 + G_2^T F_2) H_1 \\ & + (G_1^T F_3 + G_2^T F_4) H_2 \end{array}\right] \qquad (4\text{-}3)$$

Before writing down the necessary derivatives of the bicubic polynomial, (4-1) is simplified by introducing reduced variables $\bar{s}$ and $\bar{t}$.

$$\bar{s} := s - s_i , \quad \bar{t} := t - t_j$$

so that (4-1) can be abbreviated (suppressing the element indices i, j) and has the form

$$f(\bar{s}, \bar{t}) = \sum_{k=0}^{3} \sum_{\ell=0}^{3} a_{k\ell} \; \bar{s}^k \; \bar{t}^\ell . \qquad (4\text{-}1)'$$

A general derivative of (4-1)′ can be given by the formula

$$D^{\alpha} f(\bar{s}, \bar{t}) = \frac{\partial^{|\alpha|} f(\bar{s}, \bar{t})}{\partial \bar{s}^{\alpha_1} \partial \bar{t}^{\alpha_2}}$$

$$= \alpha_1 ! \alpha_2 ! \sum_{k=\alpha_1}^{3} \sum_{\ell=\alpha_2}^{3} \binom{k}{\alpha_1}\binom{\ell}{\alpha_2} \bar{s}^{\,k-\alpha_1} \bar{t}^{\,\ell-\alpha_2} \qquad (4\text{-}4)$$

with the multi-index $\alpha := (\alpha_1, \alpha_2)$, $|\alpha| = \alpha_1 + \alpha_2$, $0 \le \alpha_1, \alpha_2 \le 3$.

This nice expression, however, is far from optimal with regard to CPU-time savings. Therefore, all partial derivatives of f actually used in the computations (indicated by subscripts) follow:

$$\begin{aligned} f(\bar{s}, \bar{t}) = a_{00} &+ \bar{t}\,(a_{01} + \bar{t}\,(a_{02} + \bar{t}\,a_{03})) \\ &+ \bar{s}\,((a_{10} + \bar{t}(a_{11} + \bar{t}\,(a_{12} + \bar{t}\,a_{13}))) \\ &+ \bar{s}\,((a_{20} + \bar{t}(a_{21} + \bar{t}\,(a_{22} + \bar{t}\,a_{23}))) \\ &+ \bar{s}\,(a_{30} + \bar{t}(a_{31} + \bar{t}\,(a_{32} + \bar{t}\,a_{33}))))), \end{aligned} \qquad (4\text{-}5a)$$

$$f_s(\bar{s}, \bar{t}) = a_{10} + \bar{t}\,(a_{11} + \bar{t}\,(a_{12} + \bar{t}a_{13}))$$
$$+ \bar{s}\,(2(a_{20} + \bar{t}\,(a_{21} + \bar{t}\,(a_{22} + \bar{t}a_{23})))$$
$$+ \bar{s}\cdot 3\,(a_{30} + \bar{t}\,(a_{31} + \bar{t}\,(a_{32} + \bar{t}a_{33})))), \tag{4-5b}$$

$$f_{ss}\,(\bar{s}, \bar{t}) = 2\,(a_{20} + \bar{t}\,(a_{21} + \bar{t}\,(a_{22} + \bar{t}a_{23}))$$
$$+ \bar{s}\cdot 3\,(a_{30} + \bar{t}\,(a_{31} + \bar{t}\,(a_{32} + \bar{t}a_{33})))), \tag{4-5c}$$

$$f_t(\bar{s}, \bar{t}) = a_{01} + \bar{s}\,(a_{11} + \bar{s}\,(a_{21} + \bar{s}a_{31}))$$
$$+ \bar{t}\,(2\,(a_{02} + \bar{s}\,(a_{12} + \bar{s}\,(a_{22} + \bar{s}\,a_{32})))$$
$$+ \bar{t}\cdot 3\,(a_{03} + \bar{s}\,(a_{13} + \bar{s}\,(a_{23} + \bar{s}\,a_{33})))), \tag{4-5d}$$

$$f_{tt}\,(\bar{s}, \bar{t}) = 2\,(a_{02} + \bar{s}\,(a_{12} + \bar{s}\,(a_{22} + \bar{s}\,a_{32}))$$
$$+ \bar{t}\cdot 3\,(a_{03} + \bar{s}\,(a_{13} + \bar{s}\,(a_{23} + \bar{s}\,a_{33})))), \tag{4-5e}$$

$$f_{st}\,(\bar{s}, \bar{t}) = a_{11} + \bar{t}\,(2\,a_{12} + \bar{t}\cdot 3\,a_{13})$$
$$+ \bar{s}\,(2\,(a_{21} + \bar{t}\,(2\,a_{22} + \bar{t}\cdot 3\,a_{23}))$$
$$+ \bar{s}\cdot 3\,(a_{31} + \bar{t}\,(2\,a_{32} + \bar{t}\cdot 3\,a_{33}))), \tag{4-5f}$$

$$f_{sst}\,(\bar{s}, \bar{t}) = 2\,(a_{21} + \bar{t}\,(2\,a_{22} + \bar{t}\cdot 3\,a_{23})$$
$$+ \bar{s}\cdot 3\,(a_{31} + \bar{t}\,(2\,a_{32} + \bar{t}\cdot 3\,a_{33}))). \tag{4-5g}$$

The reader is invited to verify that the function values together with the partial derivatives f_s, f_t and f_{st} are reproduced at the four corners of the rectangle (hint: use (4-5), (4-2) and (4-3).

In all covariance expressions derivatives are taken with respect to the spherical coordinates (r, φ, λ), (r', φ', λ') of the points P and Q and not only with respect to s and t. According to the chain rule for differentiation we need to know the partial derivatives of s with respect to r, r′ and of t with respect to φ, λ, φ', λ'; the latter are listed

explicitly in (Tscherning, 1976, pp. 18, 19). To find the radial derivatives of s using (2-2b) is also trivial. The result is

$$
\begin{aligned}
D_r s &= s_r = -\frac{s}{r}, \\
D_{r'} s &= s_{r'} = -\frac{s}{r'}, \\
D_r^2 s &= s_{rr} = \frac{2s}{r^2}, \\
D_r D_{r'} s &= s_{rr'} = \frac{s}{rr'}, \\
D_{r'}^2 s &= s_{r'r'} = \frac{2s}{r'^2}.
\end{aligned} \tag{4-6}
$$

Furthermore, whenever calculating derivatives of f with respect to latitude and/or longitude, it is necessary to be aware of the fact that an $|\alpha|$-th order derivative consists of a linear combination of all derivatives of f with respect to t up to the $|\alpha|$-th order,

$$
\begin{aligned}
D^{\alpha} f &= D_{\alpha_1} D_{\alpha_2} \ldots D_{\alpha_n} f \\
&= \sum_{i=1}^{n} c_i D_t^i f
\end{aligned} \tag{4-7}
$$

with α_i, $i = 1, \ldots, n$ any one of $\{\varphi, \lambda, \varphi', \lambda'\}$ and $\alpha = (\alpha_1, \ldots, \alpha_n)$. The coefficients c_i, $i = 1, \ldots, n$ consist of derivatives of t with respect to $\{\varphi, \lambda, \varphi', \lambda'\}$. General formulas are easy to derive and can be found in (Tscherning, 1976, p. 16).

On the next pages we give all basic covariance expressions in their approximated version are given. The following abbreviations will be used:

f1...	bicubic spline representation of cov (T, T),	
f2...	bicubic spline representation of cov (D_r T, $D_{r'}$ T),	
f3...	bicubic spline representation of cov (D_t^2 T, T),	(4-8a)
f4...	bicubic spline representation of cov (Δg, Δg).	

Several times the subsequent identities were used;

$$s_{rr} + \frac{2}{r}\, s_r = 0 \;,$$

$$s_r\, s_{r'} = s\; s_{rr'} \;,$$

$$\frac{s_r}{r'} + \frac{s_{r'}}{r} = -2 s_{rr'} \;,$$

$$\frac{s_r}{r'} - \frac{s_{r'}}{r} = 0 \;, \tag{4-8b}$$

$$f2 = f1_{ss}\; s_r\, s_{r'} + f1_s\; s_{rr'} \;,$$

$$f4 = f2 - 4 s_{rr'}\, (f1_s - \frac{1}{s}\, f1) \;,$$

$$f4_{ss}\; s_r\, s_{r'} + f4_s\; s_{rr'} = f2_{ss}\; s_r\, s_{r'} - 3\; f2_s\; s_{rr'} + \frac{4}{r^2 r'^2}\, f1 \;,$$

$$\mathrm{cov}\,(D_r^2\; T,\; \Delta g) = \mathrm{cov}\;(D_r\; \Delta g,\; D_{r'}\; T).$$

After having established the relations between basic quantities, the "long march" of deriving all covariance expressions begins. The results are listed below.

$$\mathrm{cov}\;(T, T) = f1$$

$$\mathrm{cov}\;(T, D_{r'}\; T) = -f1_s \frac{s}{r'}$$

$$\mathrm{cov}\;(T,\; \Delta g) = (f1_s\, s - 2\, f1) \frac{1}{r'}$$

$$\mathrm{cov}\;(T, D_{r'}\; \Delta g) = \frac{1}{r'^2}\, (-\, rr'\; f2 + s\; f1_s + 2\; f1)$$

$$\mathrm{cov}\;(T, D^2_{r'}\; T) = \frac{1}{r'}\; (r\; f2 + \frac{s}{r'}\; f1_s) \tag{4-9}$$

$$\mathrm{cov}\;(T,\; D_{\alpha_1}\; T) = f1_t\; c_1$$

$$\mathrm{cov}\;(T,\; D_{\alpha_1} D_{r'}\, T) = -f1_{st}\; \frac{s}{r'}\; c_1$$

$$\mathrm{cov}\;(T,\; D_{\alpha_1} \Delta g) = (f1_{st}\; s - 2\; f1_t) \frac{1}{r'}\; c_1$$

$$\mathrm{cov}\;(T,\; D_{\alpha_1} D_{\alpha_2}\; T) = f3\; c_2 + f1_t\; c_1$$

$$\mathrm{cov}\,(D_r T,\ D_{r'} T) = f2$$

$$\mathrm{cov}\,(D_r T,\ \Delta g) = -f2 + 2\ f1_s\ \frac{s}{rr'}$$

$$\mathrm{cov}\,(D_r T,\ D_{r'}\,\Delta g) = (f2_s s - 2\ f_2 - 2\ f1_s\ \frac{s}{rr'})\ \frac{1}{r'}$$

$$\mathrm{cov}\,(D_r T,\ D^2_{r'}\,T) = -f2_s\ \frac{s}{r'}$$

$$\mathrm{cov}\,(D_r T,\ D_{\alpha_1}\,T) = -f1_{st}\ \frac{s}{r}\ c_1 \tag{4-10}$$

$$\mathrm{cov}\,(D_r T,\ D_{\alpha_1} D_{r'} T) = f2_t\ c_1$$

$$\mathrm{cov}\,(D_r T,\ D_{\alpha_1}\,\Delta g) = (-f2_t + 2\ f1_{st}\ \frac{s}{rr'})\ c_1$$

$$\mathrm{cov}\,(D_r T,\ D_{\alpha_1} D_{\alpha_2}\,T) = -\,(f3_s\ c_2 + f1_{st}\ c_1)\ \frac{s}{r}$$

$$\mathrm{cov}\,(\Delta g, \Delta g) = f4$$

$$\mathrm{cov}\,(\Delta g,\ D_{r'}\,\Delta g) = -\ f4_s\ \frac{s}{r'}$$

$$\mathrm{cov}\,(\Delta g,\ D^2_{r'}\,T) = \mathrm{cov}\,(D_r T,\ D_{r'}\,\Delta g)$$

$$\mathrm{cov}\,(\Delta g,\ D_{\alpha_1}\,T) = (f1_{st}\ s - 2\ f1_t)\ \frac{1}{r}\ c_1 \tag{4-11}$$

$$\mathrm{cov}\,(\Delta g,\ D_{\alpha_1} D_{r'} T) = (-f2_t + 2\ f1_{st}\ \frac{s}{rr'}\ c_1$$

$$\mathrm{cov}\,(\Delta g,\ D_{\alpha_1}\,\Delta g) = f4_t\ c_1$$

$$\mathrm{cov}\,(\Delta g,\ D_{\alpha_1} D_{\alpha_2}\,T) = [\ (f3_s\ s - 2\ f3)\ c_2 + (f1_{st} s - 2\ f1_t)\ c_1]\ \frac{1}{r}$$

$$\mathrm{cov}\,(D_r\,\Delta g,\ D_{r'}\,\Delta g) = (f4_{ss}\ s + f4_s\ )\ \frac{s}{rr'}$$

$$\mathrm{cov}\,(D_r\,\Delta g,\ D^2_{r'},\ T) = (f2_{ss}\ s - f2_s\ - (f2\ \frac{1}{s} + f1_s\ \frac{1}{rr'}))\ \frac{s}{rr'}$$

$$\mathrm{cov}\,(D_r\,\Delta g,\ D_{\alpha_1}\,T) = (-r'\ f2_t + f1_{st}\ \frac{s}{r} - \frac{2}{r} f1_t\ )\ \frac{1}{r}\ c_1$$

$$\mathrm{cov}\,(D_r\,\Delta g,\ D_{\alpha_1} D_{r'} T) = (f2_{st} s - 2\ f1_{st} \frac{s}{rr'} - 2\ f2_t)\ \frac{1}{r}\ c_1 \tag{4-12}$$

$$\mathrm{cov}\,(D_r\,\Delta g,\ D_{\alpha_1}\,\Delta g) = -f4_{st}\ \frac{s}{r}\ c_1$$

$$\mathrm{cov}\,(D_r\,\Delta g,\ D_{\alpha_1} D_{\alpha_2} T) = [\ (2\ f3\ - f3_{ss}\ s^2)\ c_2\ + (-rr'\ f2_t\ + f1_{st} s + 2 f1_t) c_1]\ \frac{1}{r^2}$$

$$\mathrm{cov}\,(D_r^2\,T,\ D_{r'}^2\,T) = (f2_{ss}\,s + f2_s)\,\frac{s}{rr'}\ .$$

$$\mathrm{cov}\,(D_r^2\,T,\ D_{\alpha_1}\,T) = (r'\,f2_t + \frac{s}{r}\,f1_t)\,\frac{1}{r}\,c_1$$

$$\mathrm{cov}\,(D_r^2\,T,\ D_{\alpha_1} D_{r'}\,T) = -\,f2_{st}\,\frac{s}{r}\,c_1 \qquad (4\text{-}13)$$

$$\mathrm{cov}\,(D_r^2\,T,\ D_{\alpha_1}\,\Delta g) = (f2_{st}\,s - 2\,f2_t - 2\,f1_{st}\,\frac{s}{rr'})\,\frac{1}{r}\,c_1$$

$$\mathrm{cov}\,(D_r^2\,T,\ D_{\alpha_1} D_{\alpha_2}\,T) = [\,(f3_{ss}\,s + 2\,f3_s)\,s\,c_2 + (rr'\,f2_t + f1_{st}\,s)\,c_1]\,\frac{1}{r^2}$$

$$\mathrm{cov}\,(D_{\alpha_1}\,T,\ D_{\alpha_2}\,T) = f3\,c_2 + f1_t\,c_1$$

$$\mathrm{cov}\,(D_{\alpha_1}\,T,\ D_{\alpha_2} D_{r'}\,T) = -(f3_s\,c_2 + f1_{st}\,c_1)\,\frac{s}{r'} \qquad (4\text{-}14)$$

$$\mathrm{cov}\,(D_{\alpha_1}\,T,\ D_{\alpha_2}\,\Delta g) = [\,(f3_s\,s - 2\,f3)\,c_2 + (f1_{st}\,s - 2\,f1_t)\,c_1]\,\frac{1}{r'}$$

$$\mathrm{cov}\,(D_{\alpha_1}\,T,\ D_{\alpha_2} D_{\alpha_3}\,T) = f3_t\,c_3 + f3\,c_2 + f1_t\,c_1$$

$$\mathrm{cov}\,(D_{\alpha_1} D_r\,T,\ D_{\alpha_2} D_{r'}\,T) = f2_{tt}\,c_2 + f2_t\,c_1$$

$$\mathrm{cov}\,(D_{\alpha_1} D_r\,T,\ D_{\alpha_2}\,\Delta g) = -(f2_{tt} - 2\,f3_s\,\frac{s}{rr'})\,c_2 - (f2_t - 2\,f1_{st}\,\frac{s}{rr'})\,c_1 \qquad (4\text{-}15)$$

$$\mathrm{cov}\,(D_{\alpha_1} D_r\,T,\ D_{\alpha_2} D_{\alpha_3}\,T) = (f3_{st}\,c_3 + f3_s\,c_2 + f1_{st}\,c_1)\,s_r$$

$$\mathrm{cov}\,(D_{\alpha_1}\,\Delta g,\ D_{\alpha_2}\,\Delta g) = f4_{tt}\,c_2 + f4_t\,c_1 \qquad (4\text{-}16)$$

$$\mathrm{cov}\,(D_{\alpha_1}\,\Delta g,\ D_{\alpha_2} D_{\alpha_3}\,T) = [\,f3_{st}\,c_3 + f3_s\,c_2 + f1_{st}\,c_1)s - 2\,(f3_t c_3 + f3\,c_2 + f1_t\,c_1)]\,\frac{1}{r}$$

$$\mathrm{cov}\,(D_{\alpha_1} D_{\alpha_2}\,T,\ D_{\alpha_3} D_{\alpha_4}\,T) = f3_{tt}\,c_4 + f3_t\,c_3 + f3\,c_2 + f1_t\,c_1 \qquad (4\text{-}17)$$

All coefficients c_i, $i = 1, \ldots, 4$ occurring in the foregoing expressions are identical with the coefficients of K_1 (the partial derivatives of the covariance function with respect to t) explicitly listed in (Tscherning, 1976, pp. 17, 18, 19). The equations derived in this chapter permit the computation of all covariance expressions for second and lower order derivatives of the disturbing potential.

5. Program Guideline

The very idea behind covariance approximations, as stated in the introduction, can be sketched as follows:

a) generate a sufficiently dense network of basic covariances (here: cov (T, T), cov (D_r T, D_r, T), cov (D_t^2 T, T), cov (Δg, Δg) at the grid points of a regular rectangular grid); calculate the coefficients of the interpolating function and store this information once on some file.

b) compute all other covariances by a simple differentiation - interpolation procedure applied to the interpolation function chosen in (a).

The program is organized accordingly: it consists basically of two parts, the covariance network generation program COVNET and the covariance approximation program COVAPP.

5.1 The covariance network generation program COVNET

According to its purpose the program performs successively the following tasks:

a) it reads the parameters of the covariance function which coincide with the parameters used in Tscherning's COVAX subroutine.

b) It reads the parameters of the covariance grid and calculates the grid values. The following figure may help to make the meaning of the grid parameters more clear:

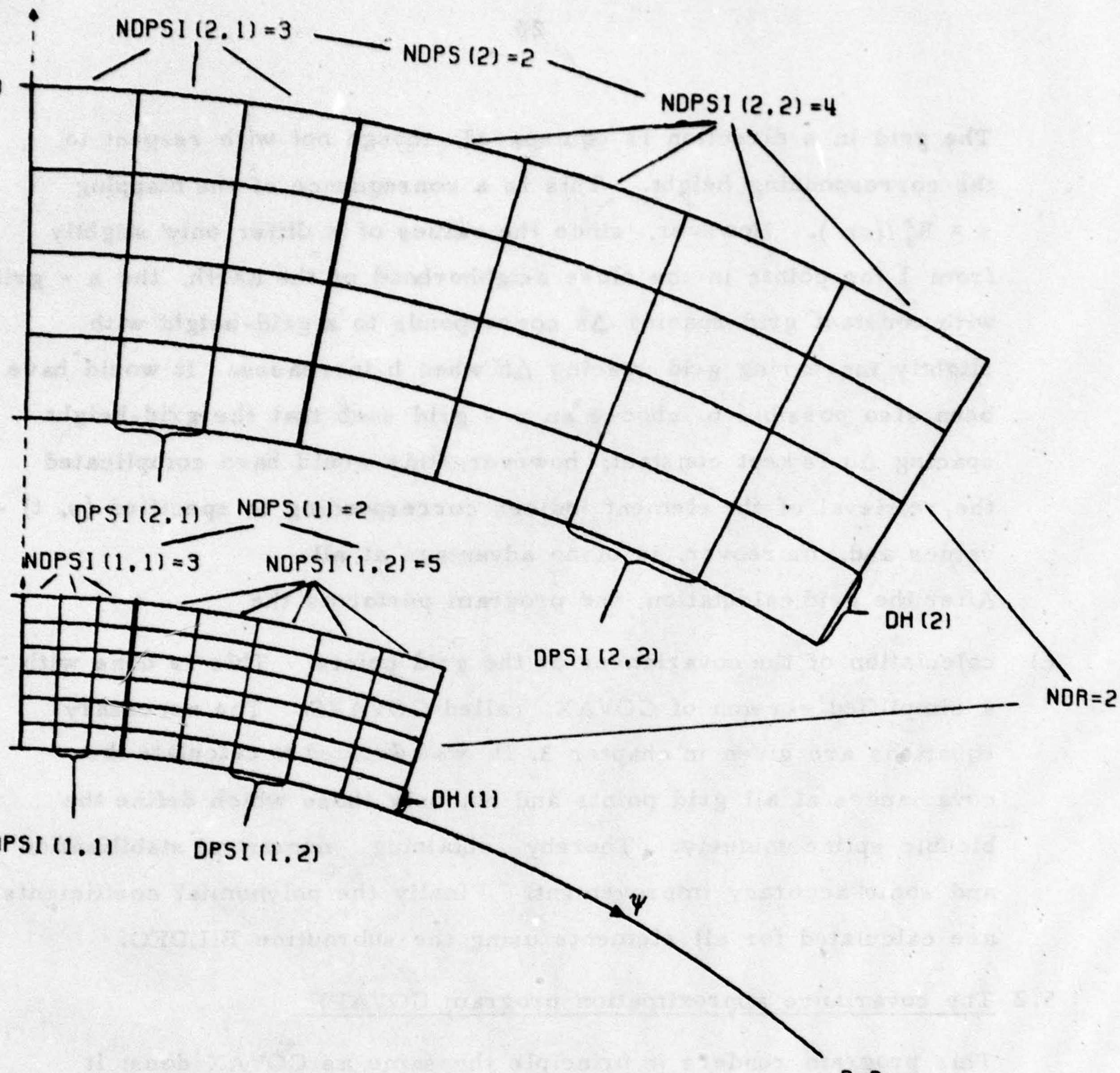

Fig. 3 grid parameter description

The grid in s direction is equispaced, though not with respect to the corresponding height. This is a consequence of the mapping $s = R_B^2/(rr')$. However, since the values of s differ only slightly from 1 for points in the close neighborhood of the Earth, the s - grid with constant grid spacing Δs corresponds to a grid-height with slightly increasing grid spacing Δh when h increases. It would have been also possible to choose an s - grid such that the grid-height spacing Δh is kept constant; however, this would have complicated the retrieval of the element indices corresponding to specified (s, t) - values and, moreover, is of no advantage at all.

After the grid calculation, the program performs the

c) calculation of the covariances at the grid points. This is done with a simplified version of COVAX, called COVAXS. The necessary equations are given in chapter 3. It was decided to calculate the covariances at all grid points and not only those which define the bicubic spline uniquely. Thereby obtaining numerical stabilization and some accuracy improvement. Finally the polynomial coefficients are calculated for all elements using the subroutine BILDEC.

5.2 The covariance approximation program COVAPP

This program renders in principle the same as COVAX does; it provides covariances between 14 different quantities which can be described as linear functionals applied on T. These quantities are described in (Tscherning, 1976); however, for the sake of completeness we list them here again:

code	quantity	description
1	ζ	height anomaly
2	$-\frac{1}{r} D_r T$	radial derivative of T divided by r
3	$\Delta g = -D_r T - \frac{2}{r} T$	free air gravity anomaly
4	$-D_r\ \Delta g$	radial component of the gravity anomaly gradient
5	$D_r^2\ T$	rr - component of the second order tensor of T
6	$\xi = -\frac{1}{r_\gamma} D\varphi T$	N-S component of the vertical deflection
7	$\eta = -\frac{1}{r\gamma \cos\varphi} D_\lambda T$	E-W component of the vertical deflection
8	$\frac{1}{r} D\varphi D_r T$	$r\varphi$ - component of the second order tensor of T
9	$\frac{1}{r \cos\varphi} D\lambda D_r T$	$r\lambda$ - component of the second order tensor of T
10	$-\frac{1}{r} D\varphi \Delta g$	N-S component of the gravity anomaly gradient
11	$-\frac{1}{r \cos\varphi} D_\lambda \Delta g$	E-W component of the gravity anomaly gradient
12	$\frac{1}{r^2} D_\varphi^2\ T$	$\varphi\varphi$ - component of the second order tensor of T
13	$\frac{1}{r^2 \cos\varphi} D\varphi D_\lambda\ T$	$\varphi\lambda$ - component of the second order tensor of T
14	$\frac{1}{r^2 \cos_\varphi^2} D_\lambda^2\ T$	$\lambda\lambda$ - component of the second order tensor of T

Note that the codes of (8, 9) and (10, 11) have been interchanged compared with (Tscherning, 1976, pp. 2, 3). This change simplified the program's logic. The codes of the linear functionals on T taken at the points P and Q are transferred to COVAPP by KR1 and KR2. The coordinate informations about P and Q are transferred by the vector CR. The (s, t) coordinates are checked for their position within the admissible ranges specified by the grid, an error message is returned if s and/or t is out of the range. Before the actual calculation of the covariance COV, all necessary polynomial coefficients corresponding to the element in

consideration are made available by the subroutine GET. The partial derivatives of the bicubic polynomial(s) combined with interpolation is rendered by the function BSFC. The calculated covariance COV is returned to the calling program.

In order to provide the user with a fast comparison between COVAPP and COVAX, we have designed a program called TEST. Detailed descriptions are integrable parts of each program.

6. Conclusions

In this report an approximation model of the spatial covariance function of the disturbing potential has been described. The idea was to generate once a basic covariance network which is chosen sufficiently dense such that it permits the calculation of all covariances (up to and including second-order derivatives) by applying a simple differentiation - interpolation procedure to an appropriate interpolation function. It turned out that the best possible function (if "best" is defined by criteria like simplicity, smoothness, localness, etc.) for such a purpose is a bicubic spline function. The procedure described here is able to provide the same covariances as the subroutine COVAX (Tscherning, 1976), with arbitrary high accuracy (depending on the grid spacing).

The goal was to cut down significantly the computation time of covariance expressions. Covariance expressions for second and lower order derivatives of the disturbing potential are available from COVAPP, a covariance approximation program at a CPU-time level between 1.8 10^{-4} sec (cov (T, T)) and 5.2 10^{-4} sec (second order derivatives) calculated on an IBM 370 system. This corresponds to a tenfold gain in calculation speed compared with COVAX, which we believe to be a significant improvement.

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APPENDIX A: Program listing

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C     C O V N E T   COMPUTES THE BASIC NETWORK OF COVARIANCES USING A C
C                   SIMPLIFIED VERSION OF C.C. TSCHERNINGS COVAX, CALLEDC
C                   COVAXS, AND CALCULATES THE POLYNOMIAL COEFFICIENTS C
C                   FOR ALL ELEMENTS OF THE GRID.                     C
C                                                                     C
C     THE PROGRAM IS DESIGNED FOR VARIOUS SOURCES OF GEODETIC DATA :  C
C     SURFACE, AIRBORNE AND SATELLITE DATA. THE DIFFERENT ALTITUDES   C
C     ABOVE THE EARTH TO WHICH THE DATA REFER MAKE IT NECESSARY TO    C
C     ADOPT A NON-UNIFORM GRID. THE GRID USED FOR HANDLING SURFACE    C
C     AND AIRBORNE DATA ONLY WILL BE VERY DENSE (ESPECIALLY FOR SMALL C
C     SPHERICAL DISTANCES) COMPARED WITH THE GRID USED FOR SATELLITE/ C
C     SURFACE (AIRBORNE) DATA OR FOR SATELLITE/SATELLITE DATA. IN ORDERC
C     TO BE APPLICABLE FOR ALL DIFFERENT KINDS OF DATA SIMULTANEOUSLY, C
C     THE PROGRAM HAS BEEN DESIGNED IN SUCH A WAY THAT THE USER CAN   C
C     DEFINE 3 RANGES IN HEIGHT AND 5 RANGES IN SPHERICAL DISTANCE,   C
C     EACH WITH VARIABLE GRID SPACING.                                C
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C       **********          INPUT          **********                 C
C                                                                     C
C     COVARIANCE MODEL PARAMETERS:                                    C
C     ----------------------------                                    C
C                                                                     C
C     RBRE2 ...                 SQUARE OF THE RATIO BETWEEN THE RADIUS OF C
C                               THE BJERHAMMAR SPHERE AND THE MEAN RADIUS C
C                               OF THE EARTH (RE)                     C
C     RE ...                    MEAN RADIUS OF THE EARTH              C
C     A                         SCALING FACTOR OF THE GRAVITY ANOMALY DE- C
C                               GREEE VARIANCE MODEL, UNIT ... (M/SEC)**4 C
C     KI(3), KI(4) ...          INTEGERS K2 AND K3 IN FORMULA (17) IN C
C                               TSCHERNING (1976)                     C
C     KI(5) ...                 NUMBER OF DEGREE VARIANCE MODEL AS SPECI- C
C                               FIED IN TSCHERNING (1976); POSSIBLE NUM- C
C                               BERS : 1, 2, 3                        C
C     N ...                     DEGREE VARIANCES UP TO AND INCLUDING DE- C
C                               GREEE  N  ARE EITHER SET EQUAL TO ZERO OR C
C                               REPLACED BY EMPIRICAL DEGREE VARIANCES. C
C                               IN THE FIRST CASE THE LOGICAL VARIABLE C
C                               LOCAL  MUST BE .TRUE., IN THE SECOND CASE C
C                               .FALSE.                               C
C     LOCAL ...                 LOGICAL VARIABLE AS SPECIFIED ABOVE   C
C     SIGMA0(I),I=1,N+1 ...     EMPIRICAL ANOMALY DEGREE VARIANCES, UNITS C
C                               OF MGAL**2 REPLACING THE MODEL DEGREE C
C                               VARIANCES UP TO AND INCLUDING DEGREE N. C
C                               (SIGMA(K+1) CORRESPONDS TO THE DEGREE C
C                               VARIANCE OF DEGREE K)                 C
C                                                                     C
C     COVARIANCE GRID PARAMETERS :                                    C
C     ----------------------------                                    C
C                                                                     C
C     C(MH,MD,K,16) ...         4-DIM. ARRAY OF POLYNOMIAL COEFFICIENTS : C
C                               K=1 ...COV(T,T)-COEFFICIENTS,         C
C                               K=2 ...COV(DR(T),DR'(T))-COEFFICIENTS, C
C                               K=3 ...COV(DT(T),DT(T))-COEFFICIENTS, C
C                               K=4 ...COV(DELTA G,DELTA G)-COEFFICIENTS C
C                                                                     C
C     NDR ...                   NUMBER OF RANGES IN RADIAL DIRECTION  C
C                               (NDR.GE.1.AND.NDR.LE.3)               C
C     H0(I),I=1,NDR ...         LOWER LIMIT OF HEIGHT RANGE  I        C
C     H1(I),I=1,NDR ...         UPPER LIMIT OF HEIGHT RANGE  I        C
C     DH(I),I=1,NDR ...         STARTING GRID SPACING IN HEIGHT, RANGE  I C
C                                                                     C
C     S0(I),I=1,NDR ...         LOWER LIMITS OF RADIAL RANGES IN S    C
C     S1(I),I=1,NDR ...         UPPER LIMITS OF RADIAL RANGES IN S    C
C     DS(I),I=1,NDR ...         GRID DISTANCES IN S-DIRECTION         C
C                                                                     C
C     NDPS(I),I=1,NDR ...       NUMBER OF RANGES IN SPHERICAL DISTANCE C
C                               (NDPS(I).GE.1.AND.NDPS(I).LE.5)       C
C     DPSI(I,J) ...             SPHERICAL DISTANCE GRID SPACINGS (I STANDSC
C                               FOR HEIGHT RANGES, J FOR SPH. DIST. RANGESC
```

```
C      NDPSI(I,J) ...          NUMBER OF SPHERICAL DISTANCE GRID SPACINGSC
C                              (I AND J AS ABOVE)                         C
C      PSIS(I,J) ...           SPHERICAL DISTANCE RANGE STARTING VALUES   C
C      PSI1(I) ...             SPHERICAL DISTANCE UPPER LIMIT VALUES      C
C      PS(I,K) ...             EACH ROW VECTOR CONTAINS THE COSINE OF THEC
C                              SPHERICAL DISTANCE OF THE GRID POINTS      C
C      IACC(I), JACC(I,J) ...  INTEGER VECTORS USED TO ASSIGN NUMBERS TO C
C                              GRID POINTS                                C
C                                                                         C
C      **********              OUTPUT            **********               C
C                                                                         C
C      RV(L) ...               VECTOR OF GRID VALUES IN S DIRECTION       C
C      DV(I,J) ...             EACH ROW VECTOR CONTAINS THE COSINE OF     C
C                              THE SPHERICAL DISTANCE OF THE GRID POINTS C
C                                                                         C
C                                                                         C
C      THE PROGRAM USES THE SUBROUTINES COVAXS, BILDEC AND BLOCKDATA.    C
C                                                                         C
C                                                                         C
C      IF COVNET AND TEST ARE RUN SEPARATELY, THE STATEMENT               C
C      '       CALL TEST'                                                 C
C      HAS TO BE REPLACED BY A WRITE-STATEMENT SUCH, THAT ALL QUANTI-     C
C      TIES ENCOUNTED IN THE LABELED COMMON BLOCKS /GRID/ AND /GRIPAR/    C
C      ARE STORED ON FILE 1.                                              C
C                                                                         C
C      IN CASE YOU LIKE TO CHOOSE M INSTEAD OF 3 AS THE NUMBER OF RADIALC
C      RANGES AND N INSTEAD OF 5 AS THE NUMBER OF SPHERICAL DISTANCE      C
C      RANGES, YOU HAVE TO REPLACE ALL 3 BY M, ALL 5 BY N AND ALL 6 BY    C
C      N+1 WITHIN THE LABELED COMMON BLOCK /GRIPAR/. THIS CHANGE HAS TO  C
C      BE MADE IN COVNET, TEST AND COVAPP.                                C
C                                                                         C
C      IN CASE YOU LIKE TO CHANGE THE DIMENSION OF THE GRID ARRAY FROM    C
C      C(30,50,4,16) TO C(K1,K2,4,16), YOU HAVE TO MAKE THE SAME CORREC-C
C      TIONS ALSO IN TEST, BILDEC AND GET. BESIDES THAT THE DIMENSIONS    C
C      OF RV(30) AND DV(3,50) HAVE TO BE NOW RV(K1), DV(3,K2). CHANGE     C
C      THE DIMENSION OF DV ALSO IN TEST AND OF THE CORRESPONDING ARRAY    C
C      PS IN COVAPP. CHANGE ALSO THE VALUES FOR NH AND ND IN THE DATA     C
C      STATEMENT IN COVNET.                                               C
C                                                                         C
C      REFERENCES :  H. SUENKEL (1978)                                    C
C                                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                         C
C      C O V A X S  AND  C O V A X N  DIFFER FROM  C O V A X  (TSCHER-   C
C      NING, 1976) BY THE FOLLOWING MODIFICATIONS :                       C
C   1. THE COMPUTED COVARIANCE COV IS AN ELEMENT OF THE PARAMETER LIST.  C
C                                                                         C
C   2. THE ENTRIES COVAX, COVBX AND COVCX HAVE BEEN REPLACED BY VARIAB-  C
C      LES SUCH, THAT (-1,COV) CORRESPONDS TO THE ENTRY COVAX, (0,COV)    C
C      TO THE ENTRY COVBX AND (1,COV) TO COVCX.                           C
C                                                                         C
C   3. THE CRITICAL ALTITUDE HMAX IS KEPT CONSTANT AND EQUAL TO 25 KM.   C
C      IF(KI(5)<3 OR (KI(5)=3 AND KI(4)>KI(3))) AND IF(KI(3) OR KI(4)>   C
C      200), LSUM IS .TRUE., OTHERWISE .FALSE. .                          C
C      IF THE FIRST CONDITION IS NOT FULFILLED, LSUM IS . TRUE. FOR       C
C      KI(3) OR KI(4) > 200, OTHERWISE IT IS .FALSE.                      C
C      WHEN LSUM IS .TRUE., WILL THE COVARIANCES IN ALTITUDES HIGHER      C
C      THAN 25 KM BE COMPUTED BY EVALUATING THE SUM OF THE LEGENDRE -     C
C      SERIES HAVING N2=299 TERMS.                                        C
C                                                                         C
C      ADDITIONAL SIMPLIFICATIONS FOR C O V A X S :                       C
C                                                                         C
C   1. THE INTEGER ARRAYS  K19, K21 AND K23  HAVE IDENTICALLY THE VALUE  C
C      ZERO.                                                              C
C                                                                         C
C   2. THE ARRAY  C11  HAS IDENTICALLY THE VALUE ONE.                     C
C                                                                         C
C   3. SINCE  COVAXS  MUST ONLY PROVIDE DERIVATIVES OF THE DISTURBING     C
C      POTENTIAL COVARIANCE FUNCTION  IN RADIAL DIRECTION AND WITH RE-    C
C      SPECT TO THE COSINE OF THE SPHERICAL DISTANCE BETWEEN TWO POINTS,C
C      THE COMPUTATION OF THE QUANTITIES  D(1), ... ,D(36)  (TSCHER-      C
```

```
C       NING, 1976, P. 57) IS NOT NECESSARY. FOR THE SAME REASON COVAXS  C
C       NEEDS NOT TO COMPUTE THE INTEGERS SPECIFYING THE KINDS OF DIFFER-C
C       ENTIATION AND THE DIFFERENTIATION ITSELF WITH RESPECT TO THE     C
C       LATITUDES AND LONGITUDES. THEREFORE, THE PART BETWEEN THE STATE- C
C       MENTS #78 AND #85 IS NOT NECESSARY EITHER. STATEMENT #85 HAS TO  C
C       BE REPLACED BY                                                   C
C       '85    COV=C(ND1+1)'                                             C
C       (TSCHERNING, 1976, P. 60).                                       C
C               THIS LIST GIVES ONLY THE MOST ESSENTIAL CHANGES AND PRO- C
C       VIDES THE USER A TOOL TO QUICKLY ADJUST COVAX IN ORDER TO BE     C
C       FULLY APPLICABLE FOR COVNET AND TEST, RESPECTIVELY.              C
C                                                                        C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
        IMPLICIT REAL*8(A-H,O-Z)
        LOGICAL LOCAL
        COMMON /GRID/C(30,50,4,16) /GRIPAR/RB2, S0(3), DS(3), S1(3), DPSI(
     *         3,5), PSIS(3,6), PSI1(3), DV(3,50), RE, IACC(3), JACCI(3,5)
     *         ,NDR   /COM2/DUMMY(2), DX, DY, I, J
     *         /COM3/H0(3), DH(3), H1(3), JACC(3), NDPS(3), NDPSI(3,5)
     *         /COM4/RV(30)
     *         /CMCOV/CI(12), CR(51), SIGMA0(300), SIGMA(300), KI(25), N1,
     *         LOCAL
        DATA RHOM/3.43774677ID3/,MH, MD/30,50/
5000    FORMAT(3D15.1)
5001    FORMAT(2D15.7,4I5,L2)
5002    FORMAT(12F6.2)
5003    FORMAT(16I5)
5004    FORMAT(5(D10.2,I5))
6001    FORMAT(1H ,4X,'COVARIANCE FUNCTION PARAMETERS :',//,5X,'RBRE2.....
       *',1D15.7,/,5X,'A.........',1D15.7,/,5X,'KI(3).....',I3,/,5X,'KI(4)
       *.....',I3,/,5X,'KI(5).....',I3,/,5X,'N.........',I3//)
6002    FORMAT(1H ,4X,'EMPIRICAL ANOMALY DEGREE VARIANCES IN UNITS OF MGAL
       ***2 :',//,5X,25(12F6.2/))
6003    FORMAT(1H0,4X,'CPU-TIME NEEDED FOR THE CALCULATION OF THE COVARIAN
       *CE GRID...',F10.5/)
6005    FORMAT(1H ,4X,'LEVEL...',I3/)
6006    FORMAT(1H ,4X,10D12.4)
6007    FORMAT(1H ,4X,'NUMBER OF GRID POINTS IN RADIAL DIRECTION EXCEEDS T
       *HE PRESCRIBED LIMIT IN THE DIMENSION STATEMENT'/)
6008    FORMAT(1H ,4X,'NUMBER OF GRID POINTS IN SPHERICAL DISTANCE EXCEEDS
       *THE PRESCRIBED LIMIT IN THE DIMENSION STATEMENT'/)
6009    FORMAT(1H0,4X,'NUMBER OF RADIAL RANGES (NDR) ...',I3,//5X,' I
       *       H0(I)           H1(I)            DH(I)    ',/,5X,'-------------
       *----------------------------------------'/)
6010    FORMAT(1H ,4X,I2,2X,3F15.1/)
6011    FORMAT(1H0,4X,'NUMBER OF SPHERICAL DISTANCE RANGES'//5X,' I     ND
       *PS(I)',/,5X,'---------------',/)
6012    FORMAT(1H ,4X,I2,6X,I2)
6013    FORMAT(1H0,4X,'SPHERICAL DISTANCE GRID VALUES AND NUMBER OF CORRES
       *PONDING INTERVALS'//5X,' I  J       DPSI(I,J)     NDPSI(I,J)'/)
6014    FORMAT(1H ,4X,I2,I3,F14.6,6X,I3)
6015    FORMAT(1H0,4X,'GRID POINT VALUES IN RADIAL DIRECTION'/)
6016    FORMAT(1H ,4X,8F12.1)
6017    FORMAT(1H0,4X,'GRID POINT VALUES IN SPHERICAL DISTANCE'/)
6018    FORMAT(1H0,4X,'RADIAL RANGE NUMBER ...',I2/)
6019    FORMAT(1H ,4X,8F10.6)
        RE=6371.0D3
C
C       READ AND ECHO COVARIANCE PARAMETERS
C
        READ(5,5001) RBRE2,A,KI(3),KI(4),KI(5),N,LOCAL
        WRITE(6,6001) RBRE2,A,KI(3),KI(4),KI(5),N
        IF(N.LT.2) N=2
        N1=N+1
        IF(.NOT.LOCAL) READ(5,5002) (SIGMA0(I),I=1,N1)
        IF(.NOT.LOCAL) WRITE(6,6002) (SIGMA0(I),I=1,N1)
C
C       READ AND ECHO COVARIANCE GRID PARAMETERS
C
        READ(5,5003) NDR
        READ(5,5000) ((H0(I),H1(I),DH(I)),I=1,NDR)
        READ(5,5003) (NDPS(I),I=1,NDR)
        DO 21 I=1,NDR
```

```
      NDN=NDPS(I)
 21   READ(5,5004) ((DPSI(I,J),NDPSI(I,J)),J=1,NDN)
      WRITE(6,6009) NDR
      WRITE(6,6010) ((I,H0(I),H1(I),DH(I)),I=1,NDR)
      WRITE(6,6011)
      WRITE(6,6012) ((I,NDPS(I)),I=1,NDR)
      RB2=RE*RE*RBRE2
      CI(8)=A*RB2*1.0D-10
      CI(10)=RBRE2
C
C     CALCULATE THE ARRAYS S0,DS,S1,RV,IACC,PSIS,PSI1,DV,JACC,JACCI
C
      I=0
      IAC=0
 1    I=I+1
      IAC=IAC+1
      S0(I)=RB2/(H0(I)+RE)/(H0(I)+RE)
      RV(IAC)=H0(I)+RE
      IAC1=IAC+1
      RV(IAC1)=H0(I)+DH(I)+RE
      S=RB2/RV(IAC1)/RV(IAC1)
      DS(I)=S-S0(I)
      IAC=IAC1
 2    S=S+DS(I)
      R=RB2/S
      R=DSQRT(R)
      IF(R.GT.(H1(I)+DH(I)+RE)) GOTO 19
      IAC=IAC+1
      IF(IAC.LE.MD) GOTO 18
      WRITE(6,6007)
      STOP
 18   RV(IAC)=R
      GOTO 2
 19   S1(I)=S-DS(I)
      IACC(I)=IAC
      IF(I.LT.NDR) GOTO 1
      DO 23 I=1,NDR
      PSIS(I,1)=0.0D0
      DV(I,1)=1.0D0
      JAC=1
      NDN=NDPS(I)
      PSIL=0.0D0
      NNN=0
      DO 25 J=1,NDN
      NDM=NDPSI(I,J)
      DPSI(I,J)=DPSI(I,J)/RHOM
      NNN=NNN+NDM
      JACCI(I,J)=NNN
      DO 3 K=1,NDM
      JAC=JAC+1
      IF(JAC.LE.MD) GOTO 20
      WRITE(6,6008)
      STOP
 20   PSIL=PSIL+DPSI(I,J)
 3    DV(I,JAC)=DCOS(PSIL)
 25   PSIS(I,J+1)=PSIL
      JACC(I)=JAC
 23   PSI1(I)=PSIL
      WRITE(6,6013)
      DO 22 I=1,NDR
      NDN=NDPS(I)
      DO 22 J=1,NDN
 22   WRITE(6,6014) I,J,DPSI(I,J),NDPSI(I,J)
      WRITE(6,6015)
      WRITE(6,6016) (RV(I),I=1,IAC)
      WRITE(6,6017)
      DO 24 I=1,NDR
      JAC=JACC(I)
      WRITE(6,6018) I
 24   WRITE(6,6019) (DV(I,J),J=1,JAC)
C
C     CALL COVANS FOR CALCULATION OF THE BASIC QUANTITIES OF THE COVARI-
C     ANCE FUNCTION
```

```
C
      CALL COVAXS(-1,COV)
C
C     CALCULATE ALL NECESSARY COVARIANCES AT ALL GRID POINTS
C
C
C     CALCULATE COV(T,T) AT ALL GRID POINTS
C
      IAC1=1
      JAC1=1
      IACU=0
      IRK=0
30    IRK=IRK+1
      IACL=IACU
      IACU=IACC(IRK)
      IACL=IACL+1
      JAC=JACC(IRK)
      KI(6)=1
      KI(7)=1
      CALL COVAXS(0,COV)
      DO 4 I=IACL,IACU
      CR(2)=RV(I)-RE
      CR(3)=CR(2)
      DO 4 J=1,JAC
      CR(1)=DV(IRK,J)
      CALL COVAXS(1,COV)
4     C(I,J,1,1)=COV
C
C     CALCULATE COV(DR(T),DR'(T)) AT ALL GRID POINTS
C
      KI(6)=2
      KI(7)=2
      CALL COVAXS(0,COV)
      DO 5 I=IACL,IACU
      CR(2)=RV(I)-RE
      CR(3)=CR(2)
      DO 5 J=1,JAC
      CR(1)=DV(IRK,J)
      CALL COVAXS(1,COV)
5     C(I,J,2,1)=COV/RV(I)/RV(I)
C
C     CALCULATE COV(DT(T),DT(T)) AT ALL GRID POINTS
C
      KI(6)=12
      KI(7)=1
      CALL COVAXS(0,COV)
      DO 6 I=IACL,IACU
      CR(2)=RV(I)-RE
      CR(3)=CR(2)
      DO 6 J=1,JAC
      CR(1)=DV(IRK,J)
      CALL COVAXS(1,COV)
6     C(I,J,3,1)=COV
C
C     CALCULATE COV(DELTA G,DELTA G) AT ALL GRID POINTS
C
      KI(6)=3
      KI(7)=3
      CALL COVAXS(0,COV)
      DO 7 I=IACL,IACU
      CR(2)=RV(I)-RE
      CR(3)=CR(2)
      DO 7 J=1,JAC
      CR(1)=DV(IRK,J)
      CALL COVAXS(1,COV)
7     C(I,J,4,1)=COV/RV(I)/RV(I)
C
C     CALCULATE FIRST ORDER DERIVATIVES AT ALL GRID POINTS
C
      KI(6)=6
      KI(7)=1
      CALL COVAXS(0,COV)
      DO 8 J=1,JAC,JAC1
```

```
      CR(1)=DV(IRK,J)
      DO 8 I=IACL,IACU
      CR(2)=RV(I)-RE
      CR(3)=CR(2)
      CALL COVAXS(1,COV)
8     C(I,J,1,3)=COV
      KI(6)=10
      KI(7)=2
      CALL COVAXS(0,COV)
      DO 9 J=1,JAC,JAC1
      CR(1)=DV(IRK,J)
      DO 9 I=IACL,IACU
      CR(2)=RV(I)-RE
      CR(3)=CR(2)
      CALL COVAXS(1,COV)
9     C(I,J,2,3)=COV/RV(I)/RV(I)
      KI(6)=12
      KI(7)=6
      CALL COVAXS(0,COV)
      DO 10 J=1,JAC,JAC1
      CR(1)=DV(IRK,J)
      DO 10 I=IACL,IACU
      CR(2)=RV(I)-RE
      CR(3)=CR(2)
      CALL COVAXS(1,COV)
10    C(I,J,3,3)=COV
      KI(6)=8
      KI(7)=3
      CALL COVAXS(0,COV)
      DO 11 J=1,JAC,JAC1
      CR(1)=DV(IRK,J)
      DO 11 I=IACL,IACU
      CR(2)=RV(I)-RE
      CR(3)=CR(2)
      CALL COVAXS(1,COV)
11    C(I,J,4,3)=COV/RV(I)/RV(I)
      KI(6)=2
      KI(7)=1
      CALL COVAXS(0,COV)
      DO 12 I=IACL,IACU,IAC1
      CR(2)=RV(I)-RE
      CR(3)=CR(2)
      S=RB2/RV(I)/RV(I)
      DO 12 J=1,JAC
      CR(1)=DV(IRK,J)
      CALL COVAXS(1,COV)
12    C(I,J,1,2)=COV/S
      KI(6)=2
      KI(7)=5
      CALL COVAXS(0,COV)
      DO 13 I=IACL,IACU,IAC1
      CR(2)=RV(I)-RE
      CR(3)=CR(2)
      DO 13 J=1,JAC
      CR(1)=DV(IRK,J)
      CALL COVAXS(1,COV)
13    C(I,J,2,2)=COV/RB2
      KI(6)=12
      KI(7)=2
      CALL COVAXS(0,COV)
      DO 14 I=IACL,IACU,IAC1
      CR(2)=RV(I)-RE
      CR(3)=CR(2)
      S=RB2/RV(I)/RV(I)
      DO 14 J=1,JAC
      CR(1)=DV(IRK,J)
      CALL COVAXS(1,COV)
14    C(I,J,3,2)=COV/S
      KI(6)=4
      KI(7)=3
      CALL COVAXS(0,COV)
      DO 15 I=IACL,IACU,IAC1
      CR(2)=RV(I)-RE
```

```
      CR(3)=CR(2)
      S=RB2/RV(I)/RV(I)
      DO 15 J=1,JAC
      CR(1)=DV(IRK,J)
      CALL COVAXS(1,COV)
 15   C(I,J,4,2)=COV/RB2
C
C     CALCULATE SECOND ORDER MIXED DERIVATIVES AT ALL GRID POINTS
C
      DO 16 I=IACL,IACU,IAC1
      CR(2)=RV(I)-RE
      CR(3)=CR(2)
      S=RB2/RV(I)/RV(I)
      DO 16 J=1,JAC,JAC1
      CR(1)=DV(IRK,J)
      KI(6)=2
      KI(7)=6
      CALL COVAXS(0,COV)
      CALL COVAXS(1,COV)
      C(I,J,1,4)=COV/S
      KI(6)=5
      KI(7)=10
      CALL COVAXS(0,COV)
      CALL COVAXS(1,COV)
      C(I,J,2,4)=COV/RB2
      KI(6)=10
      KI(7)=12
      CALL COVAXS(0,COV)
      CALL COVAXS(1,COV)
      C(I,J,3,4)=COV/S
      KI(6)=4
      KI(7)=8
      CALL COVAXS(0,COV)
      CALL COVAXS(1,COV)
      C(I,J,4,4)=COV/RB2
 16   CONTINUE
      IF(IRK.LT.NDR) GOTO 30
      DO 17 I=1,IACU
 17   RV(I)=RB2/RV(I)/RV(I)
C
C     CALCULATE POLYNOMIAL COEFFICIENTS FOR ALL ELEMENTS
C
      DO 26 II=1,NDR
      NDN=NDPS(II)
      NDM=JACCI(II,NDN)
      DO 26 J=1,NDM
      DY=DV(II,J+1)-DV(II,J)
      NNN=IACC(II)-1
      IF(II.GT.1) NNN=NNN-IACC(II-1)
      DO 26 NN=1,NNN
      I=NN
      IF(II.GT.1) I=I+IACC(II-1)
      DX=RV(I+1)-RV(I)
 26   CALL BILDEC
      CALL TEST
      STOP
      END



      BLOCK DATA
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /COMDAT/D1,D32,D2,D3,D10
      COMMON /COM0/K7(14), K9(14), K11(14), K13(14), K19(14), K21(14),
     *K23(14), C11(14)
      DATA D1,D32,D2,D3,D10   /1.0D0,1.5D0,2.0D0,3.0D0,1.0D-10/
      DATA K7/5*0,6*1,3*2/, K19/1,4*0,2*1,7*0/, K21/0,1,3*0,6*1,3*2/, K2
     *3/6*0,1,0,1,0,1,0,1,2/, C11/1.0D0,-1.0D9,1.0D6,2*1.0D9, 2*206264.8
     *06D0,2*-1.0D9,2*1.0D9,3*1.0D9/, K9/5*1,2,3,2,3,2,3,2,2,3/, K11/11*
     *0,2,3,3/, K13/11*1,2,3,3/
      END
```

```
      SUBROUTINE BILDEC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     GENERATION OF THE BICUBIC POLYNOMIAL COEFFICIENT VECTORS FOR THE C
C     ELEMENT WITH LOWER LEFT CORNER INDICES  M,N .                    C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C         **********          INPUT           **********               C
C                                                                      C
C     D(.,.,.,.,K), K=1,4 ... 4-DIM. ARRAY CONSISTING OF FUNCTION VALUESC
C                             (K=1), 1. DER. IN X (K=2), 1. DER. IN Y  C
C                             (K=3), 2. DER. IN XY (K=4)               C
C     M, N ...                INDICES OF THE ELEMENT'S LOWER LEFT CORNERC
C     DX, DY ...              GRID DISTANCES OF THE CORRESPONDING ELE- C
C                             MENT                                     C
C                                                                      C
C         **********          OUTPUT          **********               C
C                                                                      C
C     D(.,.,.,.,K), K=1, ... ,16                                       C
C                                                                      C
C     TRANSFER IN COMMON : D, DX, DY, M, N, D1, DS2, D2, DS            C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /GRID/D(30,50,4,16)/COM2/DUMMY(2), DX, DY, M, N /CONDAT/D1,
     *      DS2, D2, DS
      DIMENSION CC(4,4),C(16),A(16)
      EQUIVALENCE (CC(1,1),C(1))
      DO 40 J=1,4
      DO 50 K=1,3,2
      K1=K/2+M
      K11=K+1
      DO 50 L=1,3,2
      L1=L/2+N
      L11=L+1
      CC(K,L)=D(K1,L1,J,1)
      CC(K11,L)=D(K1,L1,J,2)*DX
      CC(K,L11)=D(K1,L1,J,3)*DY
   50 CC(K11,L11)=D(K1,L1,J,4)*DX*DY
      C1=D2*(C(9)-C(1))
      C2=DS2*C1
      C3=D2*(C(10)-C(2))
      C4=DS2*C3
      C5=D2*(C(11)-C(3))
      C6=DS2*C5
      C7=D2*(C(12)-C(4))
      C8=DS2*C7
      C9=C(13)+C(5)
      C10=C9+C(5)
      C11=C(14)+C(6)
      C12=C11+C(6)
      C13=C(15)+C(7)
      C14=C13+C(7)
      C15=C(16)+C(8)
      C16=C15+C(8)
      A(1)=C(1)
      A(2)=C(2)
      A(5)=C(5)
      A(6)=C(6)
      A(9)=C2-C10
      A(13)=C9-C1
      A(10)=C4-C12
      A(14)=C11-C3
      A(4)=C(4)+C(2)-D2*(C(3)-C(1))
      A(3)=C(3)-C(2)-C(1)-A(4)
      A(8)=C(8)+C(6)-D2*(C(7)-C(5))
      A(7)=C(7)-C(6)-C(5)-A(8)
      A(12)=C8-C16+A(10)-D2*(C6-C14-A(9))
      A(11)=C6-C14-A(9)-A(10)-A(12)
      A(16)=D2*(C5-C13+A(13))-C7+C15+A(14)
      A(15)=C13-C5-A(13)-A(14)-A(16)
      DO 41 II=2,16
```

```
41     D(M,N,J,II)=A(II)
40     CONTINUE
       RETURN
       END



       SUBROUTINE TEST
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C      THIS PROGRAM IS DESIGNED AS A USER EXAMPLE FOR COVAPP AND PRO-  C
C      VIDES, MOREOVER, A COMPARISON OF COVAPP AND COVAXN, A SLIGHTLY  C
C      MODIFIED VERSION OF THE ORIGINAL SUBROUTINE COVAX (TSCHERNING,  C
C      1976).                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C      THE NAMES OF PARAMETERS AND VARIABLES BETWEEN THE COMMENT CARDS C
C      C1111 - C1111 ARE IDENTICAL WITH THOSE USED IN COVAPP, BETWEEN  C
C      C2222 - C2222 WITH THOSE USED IN COVNET AND COVAXN              C
C                                                                     C
C      ADDITIONAL ARRAYS USED FOR COMPARISON PURPOSES :                C
C      ICH(.) ...              VECTOR USED FOR REORDERING THE COVAXN - C
C                              COVARIANCES (8,9,10,11) ---> (10,11,8,9) C
C      SEC(.,.), RES(.,.) ...  ARRAYS USED TO STORE ALL COVARIANCES CAL- C
C                              CULATED BY COVAPP AND COVAXN            C
C      CT(.) ...               VECTOR USED FOR COORDINATE TRANSFER TO  C
C                              COVAPP                                  C
C                                                                     C
C      IF COVNET AND TEST ARE RUN SEPARATELY, THE COMMENT STATEMENT    C
C      'C      GET COVARIANCE AND GRID PARAMETERS'                     C
C      HAS TO BE REPLACED BY A READ STATEMENT FOR THE COVARIANCE AND   C
C      GRID PARAMETERS SUCH, THAT ALL QUANTITIES ENCOUNTED IN THE      C
C      LABELED COMMON BLOCKS /GRID/ AND /CRIPAR/ ARE TRANSFERRED TO TESTC
C                                                                     C
C          **********          INPUT          **********              C
C                                                                     C
C      XLATP, XLONP, HP ...    GEODETIC LATITUDE, LONGITUDE AND HEIGHT C
C                              OF THE POINT P                          C
C      XLATQ, XLONQ, HQ ...    GEODETIC LATITUDE, LONGITUDE AND HEIGHT C
C                              OF THE POINT Q                          C
C                                                                     C
C          **********          OUTPUT         **********              C
C                                                                     C
C      ALL TYPES OF COVARIANCES (1-14,1-14) CALCULATED WITH COVAPP AND C
C      WITH COVAXN AS WELL AS ITS ABSOLUTE DIFFERENCES.                C
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
       IMPLICIT REAL*8(A-H,O-Z)
C
C1111 ARRAYS AND DATA NECESSARY FOR COVARIANCE CALCULATIONS USING COVAPP
C
       COMMON /GRID/C(30,50,4,16) /GRIPAR/RB2, S0(3), DS(3), S1(3), DPSI(
     *     3,5), PSIS(3,6), PSI1(3), PS(3,50), RE, IACC(3), JACCI(3,5)
     *     , NDR    /COM1/CT(12)
       DATA RHOG,GM    /5.729577951D1,3.98D14/
C
C1111
C
C2222 ARRAYS NECESSARY FOR COVARIANCE CALCULATIONS USING COVAXN
C
       LOGICAL LOCAL
       COMMON /CMCOV/CI(12), CR(51), SIGMA0(300), SIGMA(300), KI(25), N1,
     *      LOCAL
C
C2222
C
```

```
C3333 ARRAYS AND DATA USED FOR COMPARISON PURPOSES
C
      DIMENSION ICH(14)
      DIMENSION SEC(14,14), RES(14,14)
      DATA ICH/1,2,3,4,5,6,7,10,11,8,9,12,13,14/
C
C3333
C
5003  FORMAT(16I5)
5010  FORMAT(3D20.10)
6004  FORMAT(1H1,4X,'GEODETIC COORDINATES OF P AND Q',//7X,' I          LATI
     *TUDE(P)      LONGITUDE(P)      HEIGHT(P)         LATITUDE(Q)    LONGITUDE
     *(Q)     HEIGHT(Q)'//)
6010  FORMAT(1H ,4X,I4,2(2F15.6,F12.1,5X))
6011  FORMAT(1H0,18X,'1',14X,'2',14X,'3',14X,'4',14X,'5',14X,'6',14X,'7'
     *,//)
6012  FORMAT(1H ,4X,I2,7F15.8)
6013  FORMAT(1H0,4X,'MATRIX OF COVARIANCES CALCULATED WITH COVAPP'//)
6014  FORMAT(1H0,18X,'8',14X,'9',13X,'10',13X,'11',13X,'12',13X,'13',13X
     *,'14'//)
6015  FORMAT(1H0,4X,'MATRIX OF COVARIANCES CALCULATED WITH COVAXN'//)
6018  FORMAT(1H0,10X,'RELATIVE ERRORS'//)
C
C     GET COVARIANCE AND GRID PARAMETERS
C
C
C     READ AND ECHO THE GEODETIC COORDINATES OF P AND Q
C
      NRDAT=1
444   READ(5,5010,END=40) XLATP,XLONP,HP,XLATQ,XLONQ,HQ
      WRITE(6,6004)
      WRITE(6,6010) NRDAT,XLATP,XLONP,HP,XLATQ,XLONQ,HQ
      XLATP=XLATP/RHOG
      XLONP=XLONP/RHOG
      XLATQ=XLATQ/RHOG
      XLONQ=XLONQ/RHOG
C
C     CALCULATE CR(1),... ,CR(12)
C
      CR(2)=HP
      CR(3)=HQ
      CR(4)=DSIN(XLATP)
      CR(5)=DSIN(XLATQ)
      CR(6)=DCOS(XLATP)
      CR(7)=DCOS(XLATQ)
      CR(8)=DSIN(XLONQ-XLONP)
      CR(9)=DCOS(XLONQ-XLONP)
      PSI=CR(4)*CR(5)+CR(6)*CR(7)*CR(9)
      CR(1)=PSI
      CR(12)=DARCOS(PSI)
      CR(10)=GM/(CR(2)+RE)/(CR(2)+RE)
      CR(11)=GM/(CR(3)+RE)/(CR(3)+RE)
      DO 700 I=1,12
700   CT(I)=CR(I)
C
C     CALCULATE ALL DIFFERENT TYPES OF COVARIANCES USING COVAPP
C
      DO 100 J=1,14
      DO 100 K=1,14
      CALL COVAPP(J,K,COV)
      RES(J,K)=COV
      SEC(J,K)=RES(J,K)
100   CONTINUE
C
C     PRINT ALL COVARIANCES
C
      WRITE(6,6013)
      WRITE(6,6011)
      DO 300 J=1,14
300   WRITE(6,6012) J,(RES(J,K),K=1,7)
      WRITE(6,6014)
      DO 400 J=1,14
400   WRITE(6,6012) J,(RES(J,K),K=8,14)
```

```
C
C       CALCULATE ALL DIFFERENT TYPES OF COVARIANCES USING COVAXN
C
        CALL COVAXN(-1,COV)
        DO 600 I=1,14
        KI(6)=I
        DO 600 J=1,14
        KI(7)=J
        CALL COVAXN(0,COV)
        CALL COVAXN(1,COV)
C
C       REORDERING OF COVARIANCES (8,9,10,11) ---> (10,11,8,9)
C
        II=ICH(I)
        JJ=ICH(J)
        RES(II,JJ)=COV
600     CONTINUE
        WRITE(6,6015)
        WRITE(6,6011)
        DO 602 J=1,14
602     WRITE(6,6012) J,(RES(J,K),K=1,7)
        WRITE(6,6014)
        DO 603 J=1,14
603     WRITE(6,6012) J, (RES(J,K),K=8,14)
        DO 567 I=1,14
        DO 567 J=1,14
        SEC(I,J)=1.0D0-DABS(SEC(I,J)/RES(I,J))
567     SEC(I,J)=DABS(SEC(I,J))
        WRITE(6,6018)
        WRITE(6,6011)
        DO 500 J=1,14
500     WRITE(6,6012) J,(SEC(J,K),K=1,7)
        WRITE(6,6014)
        DO 501 J=1,14
501     WRITE(6,6012) J,(SEC(J,K),K=8,14)
        NRDAT=NRDAT+1
        GOTO 444
40      RETURN
        END


        SUBROUTINE COVAPP(KR1,KR2,COV)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C       C O V A P P  COMPUTES APPROXIMATIONS OF A COVARIANCE BETWEEN TWO C
C                    QUANTITIES RELATED TO THE DISTURBING POTENTIAL OF  C
C                    THE EARTH.                                        C
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C       IT IS ESSENTIAL TO STATE THAT THIS PROGRAM ALONE IS NO ALTERNA- C
C       TIVE TO THE PARENT - SUBROUTINE  C O V A X ( C.C. TSCHERNING,  C
C       1976). COVAPP IS A DIFFERENTIATION - INTERPOLATION SUBROUTINE  C
C       WHICH IS ABLE TO PROVIDE THE SAME KIND OF COVARIANCES AS COVAX C
C       DOES WITH ARBITRARY HIGH ACCURACY FOR A MUCH LOWER PRICE IN    C
C       TERMS OF CPU - TIME.                                           C
C       IT IS DESIGNED FOR LARGE SCALE APPLICATIONS OF COLLOCATION IN- C
C       VOLVING THE CALCULATION OF A LARGE NUMBER OF COVARIANCES.      C
C       THE BASIC NETWORK OF COVARIANCES IS GENERATED BY C O V N E T   C
C       USING COVAXS, A SIMPLIFIED VERSION OF COVAX. THIS NET HAS TO BE C
C       GENERATED ONCE FOR ALL. AS SOON AS IT EXISTS, COVAPP CAN BE ACTI-C
C       VATED.                                                         C
C                                                                     C
C       FOR REFERENCE SEE (H. SUENKEL, 1978).                          C
C                                                                     C
```

```
C     IN ORDER TO MAKE COVAPP AS COMFORTABLE AS POSSIBLE, WE HAVE MINI-C
C     MIZED ITS DISTANCE TO COVAX RESULTING IN PARTLY IDENTICAL STATE- C
C     MENTS.                                                           C
C                                                                      C
C     A1(.) ...                POLYNOMIAL COEFFICIENTS OF COV(T,T)     C
C     A2(.) ...                  "        COV(DR(T),DR'(T))            C
C     A3(.) ...                  "        COV(DT(T),DT(T))             C
C     A4(.) ...                  "        COV(DELTA G,DELTA G)         C
C                                                                      C
C        **********            INPUT          **********               C
C                                                                      C
C     KR1, KR2 ...             INTEGERS CODING THE KIND OF OPERATOR TO C
C                              BE APPLIED ON THE COVARIANCE FUNCTION AT C
C                              THE POINTS P AND Q                      C
C     CR(1) ...                COSINE OF THE SPHERICAL DISTANCE BETWEEN C
C                              P AND Q                                 C
C     CR(2), CR(3) ...         HEIGHTS OF P AND Q (METERS)             C
C     CR(4), CR(5) ...         SINE OF THE LATITUDE OF P AND Q         C
C     CR(6), CR(7) ...         COSINE OF THE LATITUDE OF P AND Q       C
C     CR(8), CR(9) ...         SINE, COSINE OF THE LONGITUDE DIFFERENCE C
C     CR(10), CR(11) ...       REFERENCE GRAVITY AT P AND Q            C
C     CR(12) ...               SPHERICAL DISTANCE BETWEEN P AND Q      C
C                                                                      C
C     THE VECTOR CR IS TRANSFERRED IN THE LABELED COMMON BLOCK /COM1/. C
C     ALL ELEMENTS CONTAINED IN THE LABELED COMMON BLOCKS /GRIPAR/ AND C
C     /GRID/ MUST HAVE BEEN MADE AVAILABLE BEFORE COVAPP IS CALLED.   C
C     TO ALL ELEMENTS CONTAINED IN THE LABELED COMMON BLOCKS /COM0/   C
C     AND /COMDAT/ A VALUE HAS BEEN ASSIGNED IN THE BLOCKDATA SUB-    C
C     ROUTINE. THE MEANING OF K7(.), ... , K23(.) AND C11(.) IS IDENTI-C
C     CAL TO THAT IN COVAXN.                                           C
C                                                                      C
C     RB2 ...                  SQUARE OF THE RATIO BETWEEN THE RADIUS OF C
C                              THE BJERHAMMAR SPHERE AND THE MEAN RADIUS C
C                              OF THE EARTH (RE)                       C
C     RE ...                   MEAN RADIUS OF THE EARTH                C
C     NDR ...                  NUMBER OF RANGES IN RADIAL DIRECTION (ITS C
C                              MAXIMUM IS 3 ACCORDING TO THE DIMENSION C
C                              STATEMENTS)                             C
C     S0(I),I=1,NDR ...        LOWER LIMITS OF RADIAL RANGES IN S      C
C     DS(I),I=1,NDR ...        GRID DISTANCES IN S-DIRECTION           C
C     S1(I),I=1,NDR ...        UPPER LIMITS OF RADIAL RANGES IN S      C
C                                                                      C
C     DPSI(I,J) ...            SPHERICAL DISTANCE GRID SPACINGS        C
C     PSIS(I,J) ...            SPHERICAL DISTANCE RANGE STARTING VALUES C
C     PSI1(I) ...              SPHERICAL DISTANCE UPPER LIMIT VALUES   C
C     PS(I,K) ...              EACH ROW VECTOR CONTAINS THE COSINE OF THEC
C                              SPHERICAL DISTANCE OF THE GRID POINTS   C
C                                                                      C
C                                                                      C
C        **********            OUTPUT         **********               C
C                                                                      C
C     COV ...                  COMPUTED COVARIANCE                     C
C                                                                      C
C     THE PROGRAM USES THE FUNCTION BSFC, THE SUBROUTINES GET AND      C
C     BLOCKDATA.                                                       C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /AA/A1(16),A2(16),A3(16),A4(16),D(36)
     *  /GRIPAR/RB2, S0(3), DS(3), S1(3), DPSI(3,5), PSIS(3,6),
     *   PSI1(3), PS(3,50), RE, IACC(3), JACCI(3,5), NDR
     *  /COM0/K7(14), K9(14), K11(14), K13(14), K19(14), K21(14),
     *   K23(14), C11(14)   /COM1/CR(12)   /COM2/X, Y, DX, DY, IS,
     *   JT   /COMDAT/D1, D32, D2, D3
      EQUIVALENCE (T,CR(1)),(HP,CR(2)), (HQ,CR(3)),(SP,CR(4)),(SQ,CR(5))
     *        ,(CP,CR(6)),(CQ,CR(7)),(SD,CR(8)),(CD,CR(9)),(PSI,CR(12)),
     *        (SSS,D(11)),(SSC,D(29)),(SCS,D(14)),(SCC,D(32)),(CSS,D(25))
     *        ,(CSC,D(12)),(CCS,D(16)),(CCC,D(15))
 6004 FORMAT(1H ,4X,'SORRY, POINT LOCATION OUTSIDE LEGITIMATE LIMITS'/)
      R=HP+RE
      RS=HQ+RE
```

```
      KG1=KR1
      KG2=KR2
C
C     CALCULATION OF THE PROPER UNITS OF THE COVARIANCES (CR12)
C
      CR12=C11(KR1)*C11(KR2)
      IF(K19(KR1).EQ.1) CR12=CR12/CR(10)
      IF(K19(KR2).EQ.1) CR12=CR12/CR(11)
      IF(K21(KR1).EQ.0) GOTO 95
      CR12=CR12/R
      IF(KR1.LT.12) GOTO 96
      CR12=CR12/R
      IF(KR1.EQ.14) CR12=CR12/CR(6)
96    IF(K23(KR1).GE.1) CR12=CR12/CR(6)
95    IF(K21(KR2).EQ.0) GOTO 94
      CR12=CR12/RS
      IF(KR2.LT.12) GOTO 97
      CR12=CR12/RS
      IF(KR2.EQ.14) CR12=CR12/CR(7)
97    IF(K23(KR2).GE.1) CR12=CR12/CR(7)
94    CONTINUE
C
C     DETERMINATION WHETHER THE ROLE OF R AND RS HAVE TO BE INTERCHANGED
C
      IF(KR2.GE.KR1) GOTO 1
      RSS=RS
      RS=R
      R=RSS
      KG1=KR2
      KG2=KR1
1     CONTINUE
C
C     DETERMINATION OF THE NUMBER OF HORIZONTAL DERIVATIVES  NH
C
      NH=K7(KR1)+K7(KR2)
C
C     CALCULATION OF THE ELEMENT INDICES AND THE POINT POSITION RELA-
C     TIVE TO IT
C
      S=RB2/R/RS
      IF(S.LE.S0(1).AND.S.GE.S1(NDR)) GOTO 3
2     WRITE(6,6004)
      COV=9.99D40
      RETURN
3     I=0
4     I=I+1
      IF(S.LT.S1(I)) GOTO 5
      GOTO 6
5     IF(S.GT.S0(I+1)) GOTO 2
      GOTO 4
6     DX=DS(I)
      X=(S-S0(I))/DS(I)
      IS=DINT(X)
      X=X-IS
      IS=IS+1
      IF(I.GT.1) IS=IS+IACC(I-1)
      IF(PSI.LE.PSI1(I)) GOTO 7
      GOTO 2
7     J=0
8     J=J+1
      IF(PSI.LE.PSIS(I,J+1)) GOTO 9
      GOTO 8
9     JT=DINT((PSI-PSIS(I,J))/DPSI(I,J))+1
      IF(J.GT.1) JT=JT+JACCI(I,J-1)
      Y=T-PS(I,JT)
      DY=PS(I,JT+1)-PS(I,JT)
      Y=Y/DY
C
C     CALCULATION OF THE PARTIAL DERIVATIVES OF COS(PSI) WITH RESPECT TO
C     THE SPHERICAL COORDINATES OF P AND Q (C1, ... , C4), IF NECESSARY
C
      C1=D1
```

```
      IF(NH) 1091,1091,999
999   I=K9(KR1)
      J=K11(KR1)
      K=K9(KR2)
      M=K11(KR2)
      J1=K13(KR1)
      M1=K13(KR2)
      D(1)=D1
      CS=CP*SQ
      SC=SP*CQ
      SCC=SC*CD
      CC=CP*CQ
      CCS=CC*SD
      CSC=CS*CD
      D(2)=CS-SCC
      D(3)=CCS
      D(7)=SC-CSC
      D(13)=-CCS
      K6=6*K-6
      IF(NH.GT.1) GOTO 980
      C1=D(I+K6)
      GOTO 1091
980   D(4)=-T
      SCS=SC*SD
      SS=SP*SQ
      CCC=T-SS
      SSC=SS*CD
      CSS=CS*SD
      D(5)=-SCS
      D(6)=-CCC
      D(8)=CC+SSC
      D(9)=-CSS
      D(19)=-T
      D(31)=-CCC
      KM=K6+6*M
      IJ=I+J
      C1=D(IJ+KM)
      IF(NH.GT.2) GOTO 981
      C2=D(I)*D(J1)*D(K6+1)*D(6*M1-5)
      GOTO 1092
981   SSS=SS*SD
      D(10)=-D(7)
      D(17)=-SCC
      D(18)=CCS
      D(20)=-D(2)
      D(21)=-CCS
      D(26)=-SSS
      D(27)=-CSC
      D(33)=-CCS
      M6=6*M1-6
      IF(NH.GT.3) GOTO 982
      C2=D(IJ)*D(KM+1)+D(I+K6)*D(J1+M6)+D(I+M6)*D(J1+K6)
      C3=D(I)*D(J1)*D(K6+1)*D(M6+1)
      GOTO 1093
982   D(22)=T
      D(23)=SCS
      D(24)=CCC
      D(28)=-CSS
      D(30)=-CSS
      D(34)=CCC
      D(35)=SCS
      D(36)=CCC
      K61=K6+1
      M61=M6+1
      K6I=K6+I
      K6J=K6+J
      M6I=M6+I
      M6J=M6+J
      KM1=KM+1
      C2=D(IJ+K6)*D(M61)+D(I+KM)*D(J)+D(J+KM)*D(I)+D(IJ+M6)*D(K61)+D(IJ)
     **D(KM1)+D(K6I)*D(M6J)+D(M6I)*D(K6J)
      A=D(I)*D(K61)
      B=D(J)*D(M61)
```

```
      C3=D(IJ)*D(K61)*D(M51)+B*D(K6I)+D(J)*(D(M6I)*D(K61)+D(KM1)*D(I))+A
     **D(M6J)+D(K6J)*D(I)*D(M61)
      C4=A*B
      GOTO 1094
 1091 CONTINUE
      IF(NH.GT.0) KC2=(KC2-4)/2
      GOTO(901,902,903,904,905),KC1
  901 CALL GET(1)
      GOTO(11,12,13,14,15),KC2
C
C     COVARIANCES INVOLVING LE.1 HORIZONTAL DERIVATIVES
C
   11 COV=BSFC(A1,0,NH)*C1
      GOTO 1000
   12 COV=-BSFC(A1,1,NH)*S/RS*C1
      GOTO 1000
   13 COV=(BSFC(A1,1,NH)*S-BSFC(A1,0,NH)*D2)/RS*C1
      GOTO 1000
   14 CALL GET(2)
      COV=-((BSFC(A1,0,NH)*D2+BSFC(A1,1,NH)*S)/RS-BSFC(A2,0,NH)*R)/RS*C1
      GOTO 1000
   15 CALL GET(2)
      COV=(BSFC(A2,0,NH)*R+BSFC(A1,1,NH)*S/RS)/RS*C1
      GOTO 1000
  902 CALL GET(2)
      GOTO(21,22,23,24,25), KC2
   21 CALL GET(1)
      COV=-BSFC(A1,1,1)*S/R*C1
      GOTO 1000
   22 COV=BSFC(A2,0,NH)*C1
      GOTO 1000
   23 CALL GET(1)
      COV=(-BSFC(A2,0,NH)+BSFC(A1,1,NH)*D2*S/R/RS)*C1
      GOTO 1000
   24 CALL GET(1)
      COV= (BSFC(A2,0,NH)*D2+(BSFC(A1,1,NH)*D2/R/RS-BSFC(A2,1,NH))*S)/RS
     **C1
      GOTO 1000
   25 COV=-BSFC(A2,1,NH)*S/RS*C1
      GOTO 1000
  903 GOTO(31,32,33,34,35), KC2
   31 CALL GET(1)
      COV=-(BSFC(A1,0,1)*D2-BSFC(A1,1,1)*S)/R*C1
      GOTO 1000
   32 CALL GET(1)
      CALL GET(2)
      COV=(BSFC(A1,1,1)*D2*S/R/RS-BSFC(A2,0,1))*C1
      GOTO 1000
   33 CALL GET(4)
      COV=BSFC(A4,0,NH)*C1
      GOTO 1000
   34 CALL GET(4)
      COV= BSFC(A4,1,NH)*S*C1/RS
      GOTO 1000
   35 CALL GET(1)
      CALL GET(2)
      COV=(BSFC(A2,1,NH)*S-(BSFC(A2,0,NH)+BSFC(A1,1,NH)*S/R/RS)*D2)/RS*C
     *1
      GOTO 1000
  904 GOTO(41,42,43,44,45), KC2
   41 CALL GET(1)
      CALL GET(2)
      COV= (BSFC(A2,0,1)*RS-(BSFC(A1,1,1)*S+BSFC(A1,0,1)*D2)/R)/R*C1
      GOTO 1000
   42 CALL GET(1)
      CALL GET(2)
      COV=-(BSFC(A2,1,1)*S-(BSFC(A1,1,1)*S/R/RS+BSFC(A2,0,1))*D2)/R*C1
      GOTO 1000
   43 CALL GET(4)
      COV= BSFC(A4,1,1)*S/R*C1
      GOTO 1000
   44 CALL GET(4)
      COV=(BSFC(A4,2,NH)*S+BSFC(A4,1,NH))/R/RS*C1*S
```

```
      GOTO 1000
45    CALL GET(1)
      CALL GET(2)
      COV=(BSFC(A2,2,NH)*S-BSFC(A2,1,NH)-(BSFC(A2,0,NH)/S+BSFC(A1,1,NH)/
     *R/RS))*S/R/RS*C1
      GOTO 1000
905   CALL GET(2)
      GOTO(51,52,53,55,55), KG2
51    CALL GET(1)
      COV= (BSFC(A2,0,1)*RS+BSFC(A1,1,1)*S/R)/R*C1
      GOTO 1000
52    CONTINUE
      COV=-BSFC(A2,1,1)*S/R*C1
      GOTO 1000
53    CALL GET(1)
      COV= (BSFC(A2,1,1)*S-(BSFC(A1,1,1)*S/R/RS+BSFC(A2,0,1))*D2)/R*C1
      GOTO 1000
55    CONTINUE
      COV=(BSFC(A2,2,NH)*S+BSFC(A2,1,NH))*S*C1/R/RS
      GOTO 1000
1092  CONTINUE
C
C     COVARIANCES INVOLVING LE.2 HORIZONTAL DERIVATIVES
C
       IF(KG1.GT.5) GOTO 65
      CALL GET(1)
      CALL GET(3)
      GOTO(112,212,312,412,512),KG1
112   COV=BSFC(A3,0,0)*C2+BSFC(A1,0,1)*C1
      GOTO 1000
212   COV=-(BSFC(A3,1,0)*C2+BSFC(A1,1,1)*C1)*S/R
      GOTO 1000
312   COV=((BSFC(A3,1,0)*S-BSFC(A3,0,0)*D2)*C2+(BSFC(A1,1,1)*S-BSFC(A1,0
     *,1)*D2)*C1)/R
      GOTO 1000
412   CALL GET(2)
      COV=((BSFC(A2,0,1)*R*RS-BSFC(A1,0,1)*D2-BSFC(A1,1,1)*S)*C1+(BSFC(A
     *3,2,0)*S*S-BSFC(A3,0,0)*D2)*C2)/R/R
      GOTO 1000
512   CALL GET(2)
      COV= ((BSFC(A2,0,1)*RS+BSFC(A1,1,1)*S/R)*C1+(BSFC(A3,2,0)*S+BSFC(A
     *3,1,0)*D2)*S/R*C2)/R
      GOTO 1000
65    KG2=(KG2-4)/2
      KG1=(KG1-4)/2
      GOTO(60,61,62),KG1
60    CALL GET(1)
      CALL GET(3)
      GOTO(66,610,68),KG2
66    COV=BSFC(A3,0,0)*C2+BSFC(A1,0,1)*C1
      GOTO 1000
68    COV=((BSFC(A3,1,0)*S-BSFC(A3,0,0)*D2)*C2+(BSFC(A1,1,1)*S-BSFC(A1,0
     *,1)*D2)*C1)/RS
      GOTO 1000
610   COV=-(BSFC(A3,1,0)*C2+BSFC(A1,1,1)*C1)*S/RS
      GOTO 1000
61    CALL GET(2)
      GOTO(88,88,810), KG2
88    CALL GET(3)
      COV=BSFC(A2,0,1)*C1+(BSFC(A3,2,0)*S+BSFC(A3,1,0))*S/R/RS*C2
      GOTO 1000
810   CALL GET(1)
      CALL GET(3)
      COV=-((BSFC(A3,2,0)*S-BSFC(A3,1,0))*C2-BSFC(A1,1,1)*D2*C1)*S/R/RS-
     *BSFC(A2,0,1)*C1
      GOTO 1000
62    CALL GET(4)
      CALL GET(3)
      COV=BSFC(A4,0,1)*C1+(BSFC(A3,2,0)*S-BSFC(A3,1,0)*D3+BSFC(A3,0,0)*D
     *2*D2)*S/R/RS*C2
      GOTO 1000
1093  CALL GET(1)
      CALL GET(3)
```

```
C
C     COVARIANCES INVOLVING LE.3 HORIZONTAL DERIVATIVES
C
      KG1=(KG1-4)/2
      GOTO(612,812,1012), KG1
612   COV=BSFC(A1,0,1)*C1+BSFC(A3,0,0)*C2+BSFC(A3,0,1)*C3
      GOTO 1000
1012  COV=((BSFC(A1,1,1)*C1+BSFC(A3,1,0)*C2+BSFC(A3,1,1)*C3)*S-(BSFC(A1,
     *0,1)*C1+BSFC(A3,0,0)*C2+BSFC(A3,0,1)*C3)*D2)/R
      GOTO 1000
812   COV=-(BSFC(A1,1,1)*C1+BSFC(A3,1,0)*C2+BSFC(A3,1,1)*C3)*S/R
      GOTO 1000
C
C     COVARIANCES INVOLVING LE.4 HORIZONTAL DERIVATIVES
C
1004  CALL GET(1)
      CALL GET(3)
      COV=BSFC(A1,0,1)*C1+BSFC(A3,0,0)*C2+BSFC(A3,0,1)*C3+BSFC(A3,0,2)*C
     *4
1000  COV=COV*CR12
      RETURN
      END




      SUBROUTINE GET(K)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C     THIS SUBROUTINE MAPS THE POLYOMIAL COEFFICIENTS FOR A SPECIFIC   C
C     ELEMENT (IS,JT) ONTO A VECTOR                                    C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C          **********        INPUT          **********                C
C                                                                      C
C     C(.,.,K,J) ...         4-DIM. ARRAY OF FUNCTION VALUES (J=1),    C
C                            1. DER. IN X (J=2), 1. DER. IN Y (J=3),   C
C                            2. DER. IN XY (J=4)   FOR K=1, ... ,4 :   C
C                            K=1 ... COV(T,T),                         C
C                            K=2 ... COV(DR(T),DR'(T)),                C
C                            K=3 ... COV(DT(T),DT(T)),                 C
C                            K=4 ... COV(DELTA G,DELTA G)              C
C                                                                      C
C     IS, JT ...             INDICES OF THE ELEMENT'S LOWER LEFT CORNERC
C                                                                      C
C          **********        OUTPUT         **********                C
C                                                                      C
C     A1(.), A2(.), A3(.), A4(.) ... VECTOR OF POLYNOMIAL COEFFICIENTS C
C     CORRESPONDING TO THE ELEMENT (IS, JT).                           C
C     A1(.) ...              POLYNOMIAL COEFFICIENTS OF COV(T,T)       C
C     A2(.) ...                   "       COV(DR(T),DR'(T))            C
C     A3(.) ...                   "       COV(DT(T),DT(T))             C
C     A4(.) ...                   "       COV(DELTA G,DELTA G)         C
C                                                                      C
C     TRANSFER IN COMMON : C, IS, JT, A1, A2, A3, A4                   C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /GRID/C(30,50,4,16)    /COM2/DUMMY(4), IS, JT   /AA/A1(16),
     *          A2(16), A3(16), A4(16)
      GOTO(1,2,3,4), K
1     DO 10 I=1,16
10    A1(I)=C(IS,JT,K,I)
      RETURN
2     DO 20 I=1,16
```

```
20      A2(I)=C(IS,JT,K,I)
        RETURN
3       DO 30 I=1,16
30      A3(I)=C(IS,JT,K,I)
        RETURN
4       DO 40 I=1,16
40      A4(I)=C(IS,JT,K,I)
        RETURN
        END
```

```
        FUNCTION BSFC(A,IX,IY)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C       THIS FUNCTION CALCULATES ZERO, FIRST AND SECOND ORDER DERIVA-   C
C       DIVES OF A BICUBIC POLYNOMIAL                                  C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C       **********          INPUT          **********                  C
C                                                                      C
C       A(16) ...           VECTOR OF NORMALIZED POLYNOMIAL COEFF.     C
C       IX, IY ...          NUMBER OF X- AND Y-DERIVATIVES             C
C       X, Y ...            NORMALIZED COORDINATES OF THE CALCULATION  C
C                           POINT                                      C
C       DX, DY ...          GRID DISTANCES IN X- AND Y-DIRECTION       C
C                                                                      C
C       **********          OUTPUT         **********                  C
C                                                                      C
C       BSFC ...            0., 1. OR 2. ORDER DERIVATIVE              C
C                                                                      C
C       THE PROGRAM USES THE BLOCKDATA SUBROUTINE.                     C
C       TRANSFER IN COMMON : X,Y,DX,DY,D1,DS2,D2,DS                    C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION A(16)
        COMMON /COMDAT/D1,DS2,D2,DS /COM2/X,Y,DX,DY
        GOTO(2,9),IY
        BSFC=A(4)+Y*(A(8)+Y*(A(12)+Y*A(16)))
        GOTO(4,5),IX
C       IX=0, IY=0
        BSFC=A(3)+Y*(A(7)+Y*(A(11)+Y*A(15)))+BSFC*X
        BSFC=A(2)+Y*(A(6)+Y*(A(10)+Y*A(14)))+BSFC*X
        BSFC=A(1)+Y*(A(5)+Y*(A(9)+Y*A(13)))+BSFC*X
        RETURN
C       IX=1, IY=0
4       BSFC=A(3)+Y*(A(7)+Y*(A(11)+Y*A(15)))+BSFC*X*DS2
        BSFC=A(2)+Y*(A(6)+Y*(A(10)+Y*A(14)))+BSFC*X*D2
        BSFC=BSFC/DX
        RETURN
C       IX=2, IY=0
5       BSFC=A(3)+Y*(A(7)+Y*(A(11)+Y*A(15)))+BSFC*X*DS
        BSFC=BSFC*D2/DX/DX
        RETURN
2       YD2=Y*D2
        YDS2=Y*DS2
        BSFC=A(8)+YD2*(A(12)+YDS2*A(16))
        GOTO(7,8),IX
C       IX=0, IY=1
        BSFC=A(7)+YD2*(A(11)+YDS2*A(15))+BSFC*X
        BSFC=A(6)+YD2*(A(10)+YDS2*A(14))+BSFC*X
        BSFC=A(5)+YD2*(A(9)+YDS2*A(13))+BSFC*X
        BSFC=BSFC/DY
```

```
      RETURN
C     IX=1, IY=1
7     BSFC=A(7)+YD2*(A(11)+YDS2*A(15))+BSFC*X*DS2
      BSFC=A(6)+YD2*(A(10)+YDS2*A(14))+BSFC*X*D2
      BSFC=BSFC/DX/DY
      RETURN
C     IX=2, IY=1
8     BSFC=A(7)+YD2*(A(11)+YDS2*A(15))+BSFC*X*DS
      BSFC=BSFC*D2/DX/DX/DY
      RETURN
C     IX=0, IY=2
9     BSFC=A(13)+X*(A(14)+X*(A(15)+X*A(16)))
      BSFC=A(9)+X*(A(10)+X*(A(11)+X*A(12)))+BSFC*Y*DS
      BSFC=BSFC*D2/DY/DY
      RETURN
      END
```

APPENDIX B: Sample input and output

COVARIANCE FUNCTION PARAMETERS :

RBRE2..... 0.9996170D+00
A......... 0.4252800D+03
KI(3)..... 24
KI(4)..... 0
KI(5)..... 2
N......... 12

NUMBER OF RADIAL RANGES (NDR) ... 3

I	H0(I)	H1(I)	DH(I)
1	0.0	3000.0	200.0
2	100000.0	130000.0	7500.0
3	200000.0	260000.0	10000.0

NUMBER OF SPHERICAL DISTANCE RANGES

I	NDPS(I)
1	4
2	3
3	3

SPHERICAL DISTANCE GRID VALUES AND NUMBER OF CORRESPONDING INTERVALS

I	J	DPSI(I,J)	NDPSI(I,J)
1	1	0.000087	10
1	2	0.000291	17
1	3	0.000873	13
1	4	0.002909	6
2	1	0.002909	6
2	2	0.008727	8
2	3	0.017453	10
3	1	0.002909	6
3	2	0.008727	10
3	3	0.017453	15

GRID POINT VALUES IN RADIAL DIRECTION

6371000.0	6371200.0	6371400.0	6371600.1	6371800.1	6372000.2	6372200.3	6372400.4
6372600.5	6372800.7	6373000.8	6373201.0	6373401.2	6373601.5	6373801.7	6374002.0
6471000.0	6478500.0	6486026.1	6493578.6	6501157.4	6571000.0	6581000.0	6591045.8
6601137.7	6611276.2	6621461.5	6631694.0				

GRID POINT VALUES IN SPHERICAL DISTANCE

RADIAL RANGE NUMBER ... 1

1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1.000000	1.000000	1.000000	0.999999	0.999999	0.999998	0.999998	0.999997
0.999997	0.999996	0.999995	0.999994	0.999993	0.999992	0.999990	0.999989
0.999988	0.999986	0.999985	0.999983	0.999978	0.999971	0.999964	0.999957
0.999948	0.999939	0.999929	0.999918	0.999907	0.999894	0.999881	0.999867
0.999853	0.999799	0.999736	0.999665	0.999585	0.999497	0.999401	

RADIAL RANGE NUMBER ... 2

1.000000	0.999996	0.999983	0.999962	0.999932	0.999894	0.999848	0.999657
0.999391	0.999048	0.998630	0.998135	0.997564	0.996917	0.996195	0.994522
0.992546	0.990268	0.987688	0.984808	0.981627	0.978148	0.974370	0.970296
0.965926							

RADIAL RANGE NUMBER ... 3

1.000000	0.999996	0.999983	0.999962	0.999932	0.999894	0.999848	0.999657
0.999391	0.999048	0.998630	0.998135	0.997564	0.996917	0.996195	0.995396
0.994522	0.992546	0.990268	0.987688	0.984808	0.981627	0.978148	0.974370
0.970296	0.965926	0.961262	0.956305	0.951057	0.945519	0.939693	0.933580

GEODETIC COORDINATES OF P AND Q

I	LATITUDE(P)	LONGITUDE(P)	HEIGHT(P)	LATITUDE(Q)	LONGITUDE(Q)	HEIGHT(Q)
1	0.0	10.000000	0.0	0.008300	10.008300	0.0

MATRIX OF COVARIANCES CALCULATED WITH COVAPP

	1	2	3	4	5	6	7
1	30.52325824	0.21426987	127.11581014	16.73200996	17.17529704	0.16387404	0.16387404
2	0.21426987	0.00409737	2.54447658	2.32431746	2.33261572	0.02402802	0.02402802
3	127.11581014	2.54447658	1581.95783247	1475.67228664	1480.82265589	15.25781151	15.25781167
4	16.73200996	2.32431746	1475.67228664	4826.40182573	4831.04263276	56.79017356	56.79017415
5	17.17529704	2.33261572	1480.82265589	4831.04263276	4835.71201333	56.83830878	56.83830938
6	-0.16387404	-0.02402802	-15.25781167	-56.79017415	-56.83830938	36.33155897	-0.29298455
7	-0.16387404	-0.02402802	-15.25781151	-56.79017356	-56.83830878	-0.29298531	36.33155897
8	0.74216513	0.42392853	269.85641456	1843.48208669	1844.33030232	-155.76577664	10.10222374
9	0.74216512	0.42392852	269.85641173	1843.48206734	1844.33028297	10.10222701	-155.76577664
10	0.73971958	0.42356995	269.62871753	1842.63458939	1843.48208669	-155.22358957	10.09785144
11	0.73971958	0.42356994	269.62871470	1842.63457006	1843.48206734	10.09785470	-155.22358957
12	-8.37339590	-1.16227036	-737.90498870	-2435.35923653	-2437.68782287	-43.48039708	-13.29795521
13	0.06752465	0.07537930	48.00336718	489.82252074	489.97331196	-13.29791534	-13.29791580
14	-8.37339590	-1.16227036	-737.90498970	-2435.35924681	-2437.68783315	-13.29795447	-43.48039694

	8	9	10	11	12	13	14
1	-0.74216512	-0.74216513	-0.73971958	-0.73971958	-8.37339590	0.06752482	-8.37339608
2	-0.42392852	-0.42392853	-0.42356994	-0.42356995	-1.16227036	0.07537933	-1.16227039
3	-269.85641173	-269.85641456	-269.62871470	-269.62871753	-737.90498970	48.00338368	-737.90500519
4	-1843.48206734	-1843.48208669	-1842.63457006	-1842.63458939	-2435.35924681	489.82258212	-2435.35929791
5	-1844.33028297	-1844.33030232	-1843.48206734	-1843.48208669	-2437.68783315	489.97337340	-2437.68788430
6	-155.76577664	10.10222374	-155.22358957	10.09785144	43.48039694	13.29791548	13.29791594
7	10.10222701	-155.76577664	10.09785470	-155.22358957	13.29795447	13.29795461	43.48039740
8	2436.52558835	-489.89793780	2434.20104759	-489.74717920	-1441.33494051	-400.63216486	-400.63217998
9	-489.89798893	2436.52558835	-489.74723028	2434.20104759	-400.63232482	-400.63232902	-1441.33495563
10	2434.20104759	-489.74717920	2431.88459809	-489.59648582	-1440.68606913	-400.43371596	-400.43373108
11	-489.74723028	2434.20104759	-489.59653686	2431.88459809	-400.43387534	-400.43387954	-1440.68608425
12	1441.33494471	400.63234835	1440.68607333	400.43389886	1804.57983013	-250.29285276	619.40733718
13	400.63216065	400.63217578	400.43371176	400.43372688	-250.29286382	619.40531963	-250.29282491
14	400.63232482	1441.33494051	400.43387534	1440.68606913	619.40730069	-250.29291082	1804.57984450

MATRIX OF COVARIANCES CALCULATED WITH COVAXN

	1	2	3	4	5	6	7
1	30.52325824	0.21426988	127.11581061	16.73202082	17.17530790	0.16387382	0.16387383
2	0.21426988	0.00409737	2.54447825	2.32436028	2.33265854	0.02402736	0.02402736
3	127.11581061	2.54447825	1581.95889730	1475.69956067	1480.84993326	15.25738912	15.25738928
4	16.73202082	2.32436028	1475.69956067	4848.03364955	4852.69045420	56.78052556	56.78052616
5	17.17530790	2.33265854	1480.84993326	4852.69045420	4857.36406954	56.82865946	56.82866006
6	-0.16387383	-0.02402736	-15.25738928	-56.78052616	-56.82866006	36.33150765	-0.29298822
7	-0.16387382	-0.02402736	-15.25738912	-56.78052556	-56.82865946	-0.29298898	36.33150765
8	0.74214465	0.42385653	269.81055289	1842.31918045	1843.16725208	-155.76062197	10.10280101
9	0.74214464	0.42385653	269.81055006	1842.31916112	1843.16723274	10.10280428	-155.76062197
10	0.73969911	0.42349796	269.58286216	1841.47182713	1842.31918045	-155.21843567	10.09842866
11	0.73969910	0.42349796	269.58285933	1841.47180781	1842.31916112	10.09843191	-155.21843567
12	-8.37338408	-1.16223190	-737.88048797	-2424.02086256	-2426.34937197	-43.48171265	-13.29889265
13	0.06752549	0.07538361	48.00611114	500.66394010	500.81473994	-13.29885278	-13.29885324
14	-8.37338408	-1.16223190	-737.88048898	-2424.02087307	-2426.34938248	-13.29889192	-43.48171251

	8	9	10	11	12	13	14
1	-0.74214464	-0.74214465	-0.73969910	-0.73969911	-8.37338408	0.06752567	-8.37338425
2	-0.42385653	-0.42385653	-0.42349796	-0.42349796	-1.16223190	0.07538363	-1.16223193
3	-269.81055006	-269.81055289	-269.58285933	-269.58286216	-737.88048898	48.00612764	-737.88050447
4	-1842.31916112	-1842.31918045	-1841.47180781	-1841.47182713	-2424.02087307	500.66400148	-2424.02092394
5	-1843.16723274	-1843.16725208	-1842.31916112	-1842.31918045	-2426.34938248	500.81480137	-2426.34943340
6	-155.76062197	10.10280101	-155.21843567	10.09842866	43.48171251	13.29885292	13.29885338
7	10.10280428	-155.76062197	10.09843191	-155.21843567	13.29889192	13.29889206	43.48171296
8	2425.18717603	-500.73936159	2422.86271220	-500.58859437	-1441.54080001	-400.77855603	-400.77857115
9	-500.73941248	2425.18717603	-500.58864521	2422.86271220	-400.77871598	-400.77872018	-1441.54081513
10	2422.86271220	-500.58859437	2420.54633958	-500.43789240	-1440.89190899	-400.58009314	-400.58010826
11	-500.58864521	2422.86271220	-500.43794320	2420.54633958	-400.58025251	-400.58025671	-1440.89192411
12	1441.54080421	400.77873952	1440.89191320	400.58027604	1804.68553380	-250.33200810	619.33937205
13	400.77855182	400.77856695	400.58008894	400.58010406	-250.33201916	619.33735450	-250.33197935
14	400.77871598	1441.54080001	400.58025251	1440.89190899	619.33933556	-250.33206616	1804.68554818

ABSOLUTE ERRORS

	1	2	3	4	5	6	7
1	0.00000000	0.00000000	0.00000047	0.00001086	0.00001086	0.00000021	0.00000021
2	0.00000000	0.00000000	0.00000167	0.00004281	0.00004282	0.00000066	0.00000066
3	0.00000047	0.00000167	0.00106483	0.02727403	0.02727737	0.00042239	0.00042239
4	0.00001086	0.00004281	0.02727403	21.63182382	21.64782143	0.00964799	0.00964799
5	0.00001086	0.00004282	0.02727737	21.64782143	21.65205621	0.00964932	0.00964932
6	0.00000021	0.00000066	0.00042239	0.00964799	0.00964932	0.00005132	0.00000367
7	0.00000021	0.00000066	0.00042239	0.00964799	0.00964932	0.00000367	0.00005132
8	0.00002048	0.00007199	0.04586167	1.16290624	1.16305024	0.00515467	0.00057727
9	0.00002048	0.00007199	0.04586167	1.16290623	1.16305023	0.00057727	0.00515467
10	0.00002048	0.00007199	0.04585536	1.16276226	1.16290624	0.00515390	0.00057721
11	0.00002048	0.00007199	0.04585536	1.16276225	1.16290623	0.00057721	0.00515390
12	0.00001183	0.00003846	0.02450072	11.33837396	11.33845089	0.00131557	0.00093744
13	0.00000085	0.00000431	0.00274396	10.84141936	10.84142798	0.00093744	0.00093744
14	0.00001183	0.00003846	0.02450072	11.33837374	11.33845067	0.00093744	0.00131556

	8	9	10	11	12	13	14
1	0.00002048	0.00002048	0.00002048	0.00002048	0.00001183	0.00000085	0.00001183
2	0.00007199	0.00007199	0.00007199	0.00007199	0.00003846	0.00000431	0.00003846
3	0.04586167	0.04586167	0.04585536	0.04585536	0.02450072	0.00274396	0.02450072
4	1.16290623	1.16290624	1.16276225	1.16276226	11.33837374	10.84141935	11.33837397
5	1.16305023	1.16305024	1.16290623	1.16290624	11.33845067	10.84142797	11.33845090
6	0.00515467	0.00057727	0.00515390	0.00057721	0.00131556	0.00093744	0.00093744
7	0.00057727	0.00515467	0.00057721	0.00515390	0.00093744	0.00093744	0.00131557
8	11.33841232	10.84142378	11.33833540	10.84141517	0.20585950	0.14639117	0.14639117
9	10.84142355	11.33841232	10.84141493	11.33833540	0.14639116	0.14639116	0.20585950
10	11.33833540	10.84141517	11.33825851	10.84140658	0.20583986	0.14637718	0.14637718
11	10.84141493	11.33833540	10.84140634	11.33825851	0.14637717	0.14637717	0.20583987
12	0.20585950	0.14639116	0.20583987	0.14637717	0.10570368	0.03915535	0.06796513
13	0.14639117	0.14639117	0.14637718	0.14637718	0.03915535	0.06796513	0.03915534
14	0.14639116	0.20585950	0.14637717	0.20583986	0.06796513	0.03915535	0.10570368

	8	9	10	11	12	13	14
1	-0.74214464	-0.74214465	-0.73969910	-0.73969911	-8.37338408	0.06752567	-8.37338425
2	-0.42385653	-0.42385653	-0.42349796	-0.42349796	-1.16223190	0.07538363	-1.16223193
3	-269.81055006	-269.81055289	-269.58285933	-269.58286216	-737.88048898	48.00612764	-737.88050447
4	-1842.31916112	-1842.31918045	-1841.47180781	-1841.47182713	-2424.02087307	500.66400148	-2424.02092394
5	-1843.16723274	-1843.16725208	-1842.31916112	-1842.31918045	-2426.34938248	500.81480137	-2426.34943340
6	-155.76062197	10.10280101	-155.21843567	10.09842866	43.48171251	13.29885292	13.29885338
7	10.10280428	-155.76062197	10.09843191	-155.21843567	13.29889192	13.29889206	43.48171296
8	2425.18717603	-500.73936159	2422.86271220	-500.58859437	-1441.54080001	-400.77855603	-400.77857115
9	-500.73941248	2425.18717603	-500.58864521	2422.86271220	-400.77871598	-400.77872018	-1441.54081513
10	2422.86271220	-500.58859437	2420.54633958	-500.43789240	-1440.89190899	-400.58009314	-400.58010826
11	-500.58864521	2422.86271220	-500.43794320	2420.54633958	-400.58025251	-400.58025671	-1440.89192411
12	1441.54080421	400.77873952	1440.89191320	400.58027604	1804.68553380	-250.33200810	619.33937205
13	400.77855182	400.77856695	400.58008894	400.58010406	-250.33201916	619.33735450	-250.33197935
14	400.77871598	1441.54080001	400.58025251	1440.89190899	619.33933556	-250.33206616	1804.68554818

RELATIVE ERRORS

	1	2	3	4	5	6	7
1	0.00000000	0.00000000	0.00000000	0.00000065	0.00000063	0.00000130	0.00000130
2	0.00000000	0.00000064	0.00000066	0.00001842	0.00001836	0.00002760	0.00002760
3	0.00000000	0.00000066	0.00000067	0.00001848	0.00001842	0.00002768	0.00002768
4	0.00000065	0.00001842	0.00001848	0.00446198	0.00446099	0.00016992	0.00016992
5	0.00000063	0.00001836	0.00001842	0.00446099	0.00445757	0.00016980	0.00016980
6	0.00000130	0.00002760	0.00002768	0.00016992	0.00016980	0.00000141	0.00001253
7	0.00000130	0.00002760	0.00002768	0.00016992	0.00016980	0.00001253	0.00000141
8	0.00002760	0.00016986	0.00016998	0.00063122	0.00063101	0.00003309	0.00005714
9	0.00002760	0.00016986	0.00016998	0.00063122	0.00063101	0.00005714	0.00003309
10	0.00002768	0.00016998	0.00017010	0.00063143	0.00063122	0.00003320	0.00005716
11	0.00002768	0.00016998	0.00017010	0.00063143	0.00063122	0.00005716	0.00003320
12	0.00000141	0.00003309	0.00003320	0.00467751	0.00467305	0.00003026	0.00007049
13	0.00001253	0.00005714	0.00005716	0.02165408	0.02164758	0.00007049	0.00007049
14	0.00000141	0.00003309	0.00003320	0.00467751	0.00467305	0.00007049	0.00003026

	8	9	10	11	12	13	14
1	0.00002760	0.00002760	0.00002768	0.00002768	0.00000141	0.00001253	0.00000141
2	0.00016986	0.00016986	0.00016998	0.00016998	0.00003309	0.00005714	0.00003309
3	0.00016998	0.00016998	0.00017010	0.00017010	0.00003320	0.00005716	0.00003320
4	0.00063122	0.00063122	0.00063143	0.00063143	0.00467751	0.02165408	0.00467751
5	0.00063101	0.00063101	0.00063122	0.00063122	0.00467305	0.02164758	0.00467305
6	0.00003309	0.00005714	0.00003320	0.00005716	0.00003026	0.00007049	0.00007049
7	0.00005714	0.00003309	0.00005716	0.00003320	0.00007049	0.00007049	0.00003026
8	0.00467527	0.02165083	0.00467973	0.02165734	0.00014281	0.00036527	0.00036527
9	0.02165083	0.00467527	0.02165733	0.00467973	0.00036527	0.00036527	0.00014281
10	0.00467973	0.02165734	0.00468417	0.02166384	0.00014286	0.00036541	0.00036541
11	0.02165733	0.00467973	0.02166384	0.00468417	0.00036541	0.00036541	0.00014286
12	0.00014281	0.00036527	0.00014286	0.00036541	0.00005857	0.00015641	0.00010974
13	0.00036527	0.00036527	0.00036541	0.00036541	0.00015641	0.00010974	0.00015641
14	0.00036527	0.00014281	0.00036541	0.00014286	0.00010974	0.00015641	0.00005857

GEODETIC COORDINATES OF P AND Q

I	LATITUDE(P)	LONGITUDE(P)	HEIGHT(P)	LATITUDE(Q)	LONGITUDE(Q)	HEIGHT(Q)
4	47.423200	15.194300	0.0	47.378100	15.174300	200.0

MATRIX OF COVARIANCES CALCULATED WITH COVAPP

	1	2	3	4	5	6	7
1	30.48691242	0.21255218	126.03779000	14.63935685	15.07918913	-0.81577934	-0.24475256
2	0.21257887	0.00359141	2.22273237	1.06100494	1.06829046	-0.07852907	-0.02356053
3	126.04966020	2.22266260	1377.30642984	671.46003054	675.96624875	-49.77975966	-14.93507235
4	14.64119518	1.06100494	671.48110917	535.37443247	537.50680498	-88.39990664	-26.52200433
5	15.08108270	1.06829046	675.98746883	537.50680498	539.64702867	-88.55735892	-26.56924363
6	0.81574203	0.07851562	49.77279546	88.38476487	88.54219018	30.32763004	-0.97603421
7	0.24496985	0.02357848	14.94692427	26.54221799	26.58949330	-0.95969502	33.26157591
8	-2.42545405	-0.66008536	-419.80706141	-1044.64924286	-1045.97058529	-47.20445294	15.79373932
9	-0.72837135	-0.19822567	-126.06935770	-313.71139548	-314.10819869	15.75719344	-95.02556733
10	-2.41328048	-0.65891364	-419.06428683	-1043.33025153	-1044.64924286	-46.75186436	15.77917366
11	-0.72471559	-0.19787380	-125.84630020	-313.31529831	-313.71139548	15.74287162	-94.52919457
12	-6.98993709	-0.35222039	-222.25524484	208.39421747	207.68639993	60.61590392	-0.16978339
13	0.22514797	0.11793724	75.07087681	314.41114358	314.64712683	-0.15492460	28.03217324
14	-7.67265767	-0.70958833	-449.73137852	-744.23125783	-745.65414108	27.52276607	26.69523082

	8	9	10	11	12	13	14
1	2.42533657	0.72765673	2.41316358	0.72400455	-6.98894482	0.22097247	-7.65857110
2	0.66013627	0.19805606	0.65896447	0.19770449	-0.35215990	0.11746306	-0.70837451
3	419.82626167	125.95753042	419.08345312	125.73467078	-222.20976776	74.76769869	-448.94797514
4	1044.72981622	313.44296349	1043.41072316	313.04720524	208.55561914	313.73593149	-742.95817683
5	1046.05126056	313.83942717	1044.72981622	313.44296349	207.84792263	313.97096438	-744.37862610
6	-47.20000766	15.79225201	-46.74746169	15.77768772	-60.61015585	0.15477746	-28.00554953
7	15.75570958	-95.01661869	15.74138910	-94.52029268	0.13992639	-27.49662617	-26.66987687
8	-208.11935259	-314.26005784	-208.82373287	-314.02438519	624.19211038	-111.64442456	420.10429533
9	-314.12228489	744.30712946	-313.88715758	742.88916714	-111.50409381	419.03122961	424.86685067
10	-208.82373287	-314.02438519	-209.52135896	-313.78892980	623.28760632	-111.64211477	419.68635987
11	-313.88715758	742.88916714	-313.65224388	741.47861208	-111.50200565	418.62088897	424.46884786
12	-624.22080963	111.88019030	-623.31630498	111.87765681	-230.48063513	-64.75257734	28.34234056
13	111.74003665	-420.46407189	111.73772488	-420.04577851	-64.80189881	28.75388808	-247.53916370
14	-419.39008720	-425.23070587	-418.97939515	-424.83236222	29.16566545	-246.91746963	715.32195824

MATRIX OF COVARIANCES CALCULATED WITH COVAXN

	1	2	3	4	5	6	7
1	30.48691242	0.21255218	126.03779148	14.63936834	15.07920063	-0.81577837	-0.24475227
2	0.21257887	0.00359142	2.22273414	1.06101949	1.06830502	-0.07852783	-0.02356016
3	126.04966168	2.22266437	1377.30755661	671.46929535	675.97551742	-49.77897448	-14.93483678
4	14.64120668	1.06101949	671.49037428	536.07872913	538.20784114	-88.39360341	-26.52011322
5	15.08109420	1.06830502	675.99673781	538.20784114	540.35173670	-88.55105323	-26.56735178
6	0.81574106	0.07851438	49.77201039	88.37846272	88.53588557	30.32752966	-0.97605233
7	0.24496956	0.02357811	14.94668851	26.54032543	26.58760000	-0.95971318	33.26153054
8	-2.42541597	-0.66003832	-419.77710704	-1044.42182716	-1045.74307550	-47.19974786	15.79468064
9	-0.72835991	-0.19821155	-126.06036231	-313.64310184	-314.03987680	15.75813645	-95.02371829
10	-2.41324242	-0.65886663	-419.03434397	-1043.10292809	-1044.42182716	-46.74716078	15.78011471
11	-0.72470415	-0.19785968	-125.83730826	-313.24703237	-313.64310184	15.74381435	-94.52734621
12	-6.98991396	-0.35218528	-222.23288744	207.45777101	206.75002369	60.62380725	-0.16775636
13	0.22515215	0.11794427	75.07535395	314.11196946	314.34796676	-0.15289540	28.03330306
14	-7.67264720	-0.70957452	-449.72258479	-744.26130201	-745.68415763	27.52389777	26.69591441

	8	9	10	11	12	13	14
1	2.42529849	0.72764530	2.41312552	0.72399313	-6.98892168	0.22097665	-7.65856066
2	0.66008923	0.19804195	0.65891745	0.19769038	-0.35212479	0.11747009	-0.70836073
3	419.79630593	125.94854301	419.05350889	125.72568682	-222.18740877	74.77217602	-448.93919673
4	1044.50238298	313.37472829	1043.18338218	312.97899772	207.61901900	313.43702099	-742.98816962
5	1045.82373323	313.77116374	1044.50238298	313.37472829	206.91139271	313.67206794	-744.40859131
6	-47.19530302	15.79319324	-46.74275856	15.77862868	-60.61805896	0.15275018	-28.00667828
7	15.75665249	-95.01476983	15.74233175	-94.51844449	0.13789694	-27.49775680	-26.67055982
8	-207.18286440	-313.96114673	-207.88717448	-313.72546003	624.63943649	-111.52806876	420.16623426
9	-313.82312544	744.33713413	-313.58798406	742.91919939	-111.38761477	419.09326096	424.90330930
10	-207.88717448	-313.72546003	-208.58473109	-313.48999069	623.73481449	-111.52578922	419.74828196
11	-313.58798406	742.91919939	-313.35305638	741.50867148	-111.38555689	418.68290345	424.50529631
12	-624.66810583	111.76385806	-623.76348325	111.76135482	-235.84306031	-66.41909309	27.89576324
13	111.62358120	-420.52606387	111.62129971	-420.10775363	-66.46994019	28.30692040	-247.64131719
14	-419.45217168	-425.26719573	-419.04146274	-424.86884188	28.71830707	-247.01969360	715.32342499

ABSOLUTE ERRORS

	1	2	3	4	5	6	7
1	0.00000000	0.00000000	0.00000149	0.00001150	0.00001150	0.00000097	0.00000029
2	0.00000000	0.00000000	0.00000177	0.00001455	0.00001455	0.00000123	0.00000037
3	0.00000149	0.00000177	0.00112677	0.00926481	0.00926867	0.00078518	0.00023557
4	0.00001150	0.00001455	0.00926510	0.70429667	0.70103616	0.00630323	0.00189111
5	0.00001150	0.00001455	0.00926896	0.70103616	0.70470803	0.00630570	0.00189185
6	0.00000097	0.00000123	0.00078507	0.00630215	0.00630462	0.00010038	0.00001813
7	0.00000029	0.00000037	0.00023576	0.00189256	0.00189330	0.00001817	0.00004537
8	0.00003808	0.00004703	0.02995437	0.22741570	0.22750978	0.00470507	0.00094132
9	0.00001144	0.00001412	0.00899539	0.06829364	0.06832189	0.00094301	0.00184903
10	0.00003806	0.00004702	0.02994286	0.22732344	0.22741570	0.00470358	0.00094105
11	0.00001143	0.00001412	0.00899193	0.06826593	0.06829364	0.00094274	0.00184836
12	0.00002313	0.00003510	0.02235740	0.93644646	0.93637625	0.00790333	0.00202703
13	0.00000418	0.00000703	0.00447714	0.29917413	0.29916007	0.00202920	0.00112982
14	0.00001047	0.00001381	0.00879373	0.03004418	0.03001656	0.00113170	0.00068359

	8	9	10	11	12	13	14
1	0.00003808	0.00001142	0.00003806	0.00001142	0.00002313	0.00000418	0.00001045
2	0.00004704	0.00001411	0.00004702	0.00001411	0.00003511	0.00000703	0.00001378
3	0.02995574	0.00898741	0.02994423	0.00898396	0.02235900	0.00447734	0.00877841
4	0.22743324	0.06823520	0.22734097	0.06820752	0.93660014	0.29891050	0.02999278
5	0.22752733	0.06826343	0.22743324	0.06823520	0.93652992	0.29889644	0.02996521
6	0.00470463	0.00094123	0.00470313	0.00094096	0.00790311	0.00202727	0.00112875
7	0.00094292	0.00184886	0.00094265	0.00184818	0.00202945	0.00113063	0.00068295
8	0.93648818	0.29891111	0.93655839	0.29892515	0.44732611	0.11635580	0.06193893
9	0.29915945	0.03000467	0.29917352	0.03003226	0.11647904	0.06203136	0.03645864
10	0.93655839	0.29892515	0.93662787	0.29893911	0.44720817	0.11632555	0.06192209
11	0.29917352	0.03003226	0.29918750	0.03005940	0.11644875	0.06201448	0.03644845
12	0.44729620	0.11633223	0.44717827	0.11630199	5.36242518	1.66651575	0.44657732
13	0.11645545	0.06199198	0.11642517	0.06197512	1.66804138	0.44696768	0.10215350
14	0.06208448	0.03648986	0.06206759	0.03647966	0.44735838	0.10222396	0.00146676

	8	9	10	11	12	13	14
1	2.42529849	0.72764530	2.41312552	0.72399313	-6.98892168	0.22097665	-7.65856066
2	0.66008923	0.19804195	0.65891745	0.19769038	-0.35212479	0.11747009	-0.70836073
3	419.79630593	125.94854301	419.05350889	125.72568682	-222.18740877	74.77217602	-448.93919673
4	1044.50238298	313.37472829	1043.18338218	312.97899772	207.61901900	313.43702099	-742.98816962
5	1045.82373323	313.77116374	1044.50238298	313.37472829	206.91139271	313.67206794	-744.40859131
6	-47.19530302	15.79319324	-46.74275856	15.77862868	-60.61805896	0.15275018	-28.00667828
7	15.75665249	-95.01476983	15.74233175	-94.51844449	0.13789694	-27.49775680	-26.67055982
8	-207.18286440	-313.96114673	-207.88717448	-313.72546003	624.63943649	-111.52806876	420.16623426
9	-313.82312544	744.33713413	-313.58798406	742.91919939	-111.38761477	419.09326096	424.90330930
10	-207.88717448	-313.72546003	-208.58473109	-313.48999069	623.73481449	-111.52578922	419.74828196
11	-313.58798406	742.91919939	-313.35305638	741.50867148	-111.38555689	418.68290345	424.50529631
12	-624.66810583	111.76385806	-623.76348325	111.76135482	-235.84306031	-66.41909309	27.89576324
13	111.62358120	-420.52606387	111.62129971	-420.10775363	-66.46994019	28.30692040	-247.64131719
14	-419.45217168	-425.26719573	-419.04146274	-424.86884188	28.71830707	-247.01969360	715.32342499

RELATIVE ERRORS

	1	2	3	4	5	6	7
1	0.00000000	0.00000001	0.00000001	0.00000079	0.00000076	0.00000119	0.00000119
2	0.00000001	0.00000077	0.00000080	0.00001371	0.00001362	0.00001570	0.00001570
3	0.00000001	0.00000080	0.00000082	0.00001380	0.00001371	0.00001577	0.00001577
4	0.00000079	0.00001371	0.00001380	0.00131379	0.00130254	0.00007131	0.00007131
5	0.00000076	0.00001362	0.00001371	0.00130254	0.00130417	0.00007121	0.00007121
6	0.00000119	0.00001570	0.00001577	0.00007131	0.00007121	0.00000331	0.00001857
7	0.00000119	0.00001570	0.00001577	0.00007131	0.00007121	0.00001893	0.00000136
8	0.00001570	0.00007126	0.00007136	0.00021774	0.00021756	0.00009968	0.00005960
9	0.00001570	0.00007126	0.00007136	0.00021774	0.00021756	0.00005984	0.00001946
10	0.00001577	0.00007136	0.00007146	0.00021793	0.00021774	0.00010062	0.00005964
11	0.00001577	0.00007136	0.00007146	0.00021793	0.00021774	0.00005988	0.00001955
12	0.00000331	0.00009967	0.00010060	0.00451391	0.00452903	0.00013037	0.01208315
13	0.00001857	0.00005960	0.00005964	0.00095244	0.00095168	0.01327181	0.00004030
14	0.00000136	0.00001946	0.00001955	0.00004037	0.00004025	0.00004112	0.00002561

	8	9	10	11	12	13	14
1	0.00001570	0.00001570	0.00001577	0.00001577	0.00000331	0.00001893	0.00000136
2	0.00007126	0.00007126	0.00007136	0.00007136	0.00009970	0.00005984	0.00001946
3	0.00007136	0.00007136	0.00007146	0.00007146	0.00010063	0.00005988	0.00001955
4	0.00021774	0.00021774	0.00021793	0.00021793	0.00451115	0.00095365	0.00004037
5	0.00021756	0.00021756	0.00021774	0.00021774	0.00452624	0.00095289	0.00004025
6	0.00009968	0.00005960	0.00010062	0.00005964	0.00013038	0.01327181	0.00004030
7	0.00005984	0.00001946	0.00005988	0.00001955	0.01471712	0.00004112	0.00002561
8	0.00452010	0.00095206	0.00450513	0.00095282	0.00071613	0.00104329	0.00014742
9	0.00095327	0.00004031	0.00095403	0.00004042	0.00104571	0.00014801	0.00008580
10	0.00450513	0.00095282	0.00449040	0.00095358	0.00071698	0.00104304	0.00014752
11	0.00095403	0.00004042	0.00095479	0.00004054	0.00104546	0.00014812	0.00008586
12	0.00071605	0.00104088	0.00071690	0.00104063	0.02273726	0.02509091	0.01600879
13	0.00104329	0.00014742	0.00104304	0.00014752	0.02509467	0.01579005	0.00041251
14	0.00014801	0.00008580	0.00014812	0.00008586	0.01557746	0.00041383	0.00000205

GEODETIC COORDINATES OF P AND Q

1	LATITUDE(P)	LONGITUDE(P)	HEIGHT(P)	LATITUDE(Q)	LONGITUDE(Q)	HEIGHT(Q)
2	47.423200	15.194300	0.0	47.378100	15.174300	5420.0

MATRIX OF COVARIANCES CALCULATED WITH COVAPP

	1	2	3	4	5	6	7
1	29.83421628	0.20131369	119.20603019	11.77841708	12.19540993	-0.63385028	-0.19016966
2	0.20199962	0.00289748	1.78553365	0.64144077	0.64733300	-0.03729432	-0.01118915
3	119.51052548	1.78401594	1100.87004645	405.03634049	408.66191756	-23.56509991	-7.07007174
4	11.81854926	0.64144077	405.38091700	383.54921791	384.84095531	-26.74096337	-8.02290379
5	12.23696291	0.64733300	409.00957846	384.84095531	386.13856741	-26.81585824	-8.04537398
6	0.63434059	0.03719643	23.52324161	26.67077421	26.74547250	25.29864494	-0.23798896
7	0.19049444	0.01117020	7.06410214	8.00931590	8.03174803	-0.22480779	26.00009667
8	-1.15281849	-0.19928792	-126.72039872	-248.84877308	-249.24790402	-39.52749255	2.36735281
9	-0.34619495	-0.05984678	-38.05452729	-74.73005547	-74.84991573	2.34520439	-46.67022075
10	-1.14335203	-0.19873283	-126.36935373	-248.45075797	-248.84877308	-39.14995306	2.36380123
11	-0.34335215	-0.05968008	-37.94910739	-74.61053030	-74.73005547	2.34184952	-46.28221329
12	-5.83559562	-0.29419353	-185.79844001	-140.66783305	-141.25903000	19.63795888	1.50330618
13	0.05494339	0.01763445	11.22759745	33.74854495	33.78384031	1.50830580	7.31211768
14	-6.00252033	-0.34764727	-219.83153785	-242.93418447	-243.63236927	6.89773010	6.52158376

	8	9	10	11	12	13	14
1	1.14899249	0.34472416	1.13955744	0.34189343	-5.81575238	0.05163560	-5.97190457
2	0.19930329	0.05979557	0.19874816	0.05962902	-0.29418452	0.01743958	-0.34705258
3	126.62245111	37.98964642	126.27167745	37.88440626	-185.63477683	11.09486477	-219.26895566
4	248.86796668	74.66611155	248.46992088	74.54668865	-140.65052990	33.59238848	-242.51862157
5	249.26712841	74.78586924	248.86796668	74.66611155	-141.24170881	33.62729260	-243.21561207
6	-39.42678230	2.36132115	-39.05020473	2.35777861	-19.58831062	-1.50317554	-7.28724669
7	2.33922916	-46.55131188	2.33588283	-46.16429300	-1.50814943	-6.87426858	-6.49940165
8	140.95617943	-33.73731328	140.36780139	-33.70207456	178.60212640	9.04717610	70.56215276
9	-33.63863643	243.07655168	-33.60372739	242.38185209	9.09205089	69.88395207	65.61778320
10	140.36780139	-33.70207456	139.78504255	-33.66688864	178.30980400	9.02474375	70.45340293
11	-33.60372739	242.38185209	-33.56836823	241.69292733	9.06954431	69.78136524	65.52079063
12	-178.59980078	-9.01005667	-178.30748414	-8.98767952	93.80117346	-14.61518849	46.84957215
13	-9.05492408	-70.62258206	-9.03247252	-70.51373909	-14.60341648	47.00000709	-19.11526914
14	-69.94380056	-65.67397815	-69.84112587	-65.57690251	47.15019304	-18.92296654	195.74252065

MATRIX OF COVARIANCES CALCULATED WITH COVAXN

	1	2	3	4	5	6	7
1	29.83421628	0.20131369	119.20603064	11.77841730	12.19541015	-0.63385026	-0.19016965
2	0.20199962	0.00289748	1.78553369	0.64144107	0.64733330	-0.03729432	-0.01118915
3	119.51052592	1.78401597	1100.87006826	405.03653093	408.66210840	-23.56510119	-7.07007212
4	11.81854948	0.64144107	405.38110761	383.51118989	384.79976279	-26.74087419	-8.02287704
5	12.23696314	0.64733330	409.00976947	384.79976279	386.10032160	-26.81576907	-8.04534722
6	0.63434057	0.03719643	23.52324289	26.67068526	26.74538356	25.29864352	-0.23798917
7	0.19049443	0.01117020	7.06410252	8.00928919	8.03172132	-0.22480801	26.00009590
8	-1.15281855	-0.19928726	-126.71997551	-248.84610605	-249.24523567	-39.52744371	2.36736823
9	-0.34619497	-0.05984658	-38.05440020	-74.72925455	-74.84911441	2.34521982	-46.67021866
10	-1.14335209	-0.19873217	-126.36893090	-248.44809119	-248.84610605	-39.14990425	2.36381664
11	-0.34335217	-0.05967988	-37.94898041	-74.60972946	-74.72925455	2.34186494	-46.28221121
12	-5.83559529	-0.29419316	-185.79820837	-140.78912100	-141.38031723	19.63799008	1.50331145
13	0.05494344	0.01763457	11.22767065	33.71186837	33.74716396	1.50831108	7.31212546
14	-6.00252015	-0.34764725	-219.83152799	-242.94435457	-243.64253934	6.89773790	6.52159020

	8	9	10	11	12	13	14
1	1.14899255	0.34472418	1.13955750	0.34189345	-5.81575206	0.05163565	-5.97190439
2	0.19930263	0.05979537	0.19874749	0.05962882	-0.29418415	0.01743970	-0.34705257
3	126.62202823	37.98951954	126.27125495	37.88427950	-185.63454536	11.09493785	-219.26894583
4	248.86529945	74.66531132	248.46725388	74.54588849	-140.77183669	33.55574089	-242.52877428
5	249.26445985	74.78506861	248.86529945	74.66531132	-141.36301488	33.59064524	-243.22576474
6	-39.42673359	2.36133652	-39.03015604	2.35779398	-19.58834175	-1.50318080	-7.28725444
7	2.33924454	-46.55130979	2.33589821	-46.16429092	-1.50815470	-6.87427636	-6.49940806
8	141.07747644	-33.70066820	140.48909913	-33.66542925	178.60369548	9.04735537	70.56264810
9	-33.60195757	243.08671307	-33.56704830	242.39201351	9.09223057	69.88444798	65.61822347
10	140.48909913	-33.66542925	139.90634160	-33.63024316	178.31137262	9.02492294	70.45389815
11	-33.56704830	242.39201351	-33.53218897	241.70308955	9.06972391	69.78186104	65.52123080
12	-178.60136982	-9.01023569	-178.30905272	-8.98785847	93.78484977	-14.62023544	46.84820541
13	-9.03510351	-70.62307782	-9.03265187	-70.51423474	-14.60846806	46.99863921	-19.11564082
14	-69.94429690	-65.67441880	-69.84162210	-65.57734306	47.14882402	-18.92333835	195.74262999

ABSOLUTE ERRORS

	1	2	3	4	5	6	7
1	0.00000000	0.00000000	0.00000044	0.00000022	0.00000022	0.00000002	0.00000001
2	0.00000000	0.00000000	0.00000003	0.00000030	0.00000030	0.00000000	0.00000000
3	0.00000045	0.00000003	0.00002181	0.00019045	0.00019084	0.00000128	0.00000038
4	0.00000022	0.00000030	0.00019061	0.03802802	0.04119252	0.00008918	0.00002676
5	0.00000022	0.00000030	0.00019100	0.04119252	0.03824581	0.00008918	0.00002676
6	0.00000002	0.00000000	0.00000128	0.00008895	0.00008894	0.00000143	0.00000022
7	0.00000001	0.00000000	0.00000038	0.00002671	0.00002671	0.00000022	0.00000077
8	0.00000006	0.00000066	0.00042321	0.00266703	0.00266836	0.00004883	0.00001541
9	0.00000002	0.00000020	0.00012709	0.00080092	0.00080132	0.00001543	0.00000209
10	0.00000006	0.00000066	0.00042283	0.00266679	0.00266703	0.00004881	0.00001541
11	0.00000002	0.00000020	0.00012698	0.00080084	0.00080092	0.00001542	0.00000208
12	0.00000033	0.00000036	0.00023164	0.12128795	0.12128723	0.00003120	0.00000526
13	0.00000005	0.00000011	0.00007320	0.03667658	0.03667635	0.00000527	0.00000778
14	0.00000018	0.00000002	0.00000986	0.01017010	0.01017007	0.00000780	0.00000644

	8	9	10	11	12	13	14
1	0.00000006	0.00000002	0.00000006	0.00000002	0.00000033	0.00000005	0.00000018
2	0.00000066	0.00000020	0.00000066	0.00000020	0.00000036	0.00000011	0.00000002
3	0.00042288	0.00012687	0.00042250	0.00012676	0.00023148	0.00007308	0.00000984
4	0.00266723	0.00080023	0.00266699	0.00080016	0.12130679	0.03664759	0.01015271
5	0.00266856	0.00080063	0.00266723	0.00080023	0.12130607	0.03664736	0.01015268
6	0.00004871	0.00001538	0.00004869	0.00001537	0.00003112	0.00000525	0.00000775
7	0.00001539	0.00000208	0.00001539	0.00000207	0.00000526	0.00000778	0.00000641
8	0.12129701	0.03664508	0.12129774	0.03664531	0.00156908	0.00017928	0.00049534
9	0.03667886	0.01016139	0.03667909	0.01016142	0.00017968	0.00049591	0.00044027
10	0.12129774	0.03664531	0.12129904	0.03664548	0.00156862	0.00017920	0.00049523
11	0.03667909	0.01016142	0.03667926	0.01016222	0.00017960	0.00049580	0.00044017
12	0.00156904	0.00017902	0.00156837	0.00017895	0.01632369	0.00504695	0.00136674
13	0.00017943	0.00049577	0.00017935	0.00049565	0.00505158	0.00136788	0.00037168
14	0.00049634	0.00044064	0.00049622	0.00044055	0.00136902	0.00037182	0.00010934

	8	9	10	11	12	13	14
1	1.14899255	0.34472418	1.13955750	0.34189345	-5.81575206	0.05163565	-5.97190439
2	0.19930263	0.05979537	0.19874749	0.05962882	-0.29418415	0.01743970	-0.34705257
3	126.62202823	37.98951954	126.27125495	37.88427950	-185.63454536	11.09493785	-219.26894583
4	248.86529945	74.66531132	248.46725388	74.54588849	-140.77183669	33.55574089	-242.52877428
5	249.26445985	74.78506861	248.86529945	74.66531132	-141.36301488	33.59064524	-243.22576474
6	-39.42673359	2.36133652	-39.05015604	2.35779398	-19.58834175	-1.50318080	-7.28725444
7	2.33924454	-46.55130979	2.33589821	-46.16429092	-1.50815470	-6.87427636	-6.49940806
8	141.07747644	-33.70066820	140.48909913	-33.66542925	178.60369548	9.04735537	70.56264810
9	-33.60195757	243.08671307	-33.56704830	242.39201351	9.09223057	69.88444798	65.61822347
10	140.48909913	-33.66542925	139.90634160	-33.63024316	178.31137262	9.02492294	70.45389815
11	-33.56704830	242.39201351	-33.53218897	241.70308955	9.06972391	69.78186104	65.52123080
12	-178.60136982	-9.01023569	-178.30905272	-8.98785847	93.78484977	-14.62023544	46.84820541
13	-9.05510351	-70.62307782	-9.03265187	-70.51423474	-14.60846806	46.99863921	-19.11564082
14	-69.94429690	-65.67441880	-69.84162210	-65.57734306	47.14882402	-18.92333835	195.74262999

RELATIVE ERRORS

	1	2	3	4	5	6	7
1	0.00000000	0.00000000	0.00000000	0.00000002	0.00000002	0.00000003	0.00000003
2	0.00000000	0.00000002	0.00000002	0.00000047	0.00000046	0.00000005	0.00000005
3	0.00000000	0.00000002	0.00000002	0.00000047	0.00000047	0.00000005	0.00000005
4	0.00000002	0.00000047	0.00000047	0.00009916	0.00010705	0.00000334	0.00000334
5	0.00000002	0.00000046	0.00000047	0.00010705	0.00009906	0.00000333	0.00000333
6	0.00000003	0.00000005	0.00000005	0.00000334	0.00000333	0.00000006	0.00000091
7	0.00000003	0.00000005	0.00000005	0.00000334	0.00000333	0.00000096	0.00000003
8	0.00000005	0.00000333	0.00000334	0.00001072	0.00001071	0.00000124	0.00000651
9	0.00000005	0.00000333	0.00000334	0.00001072	0.00001071	0.00000658	0.00000004
10	0.00000005	0.00000334	0.00000335	0.00001073	0.00001072	0.00000125	0.00000652
11	0.00000005	0.00000334	0.00000335	0.00001073	0.00001072	0.00000659	0.00000004
12	0.00000006	0.00000124	0.00000125	0.00086149	0.00085788	0.00000159	0.00000350
13	0.00000091	0.00000651	0.00000652	0.00108794	0.00108680	0.00000350	0.00000106
14	0.00000003	0.00000004	0.00000004	0.00004186	0.00004174	0.00000113	0.00000099

	8	9	10	11	12	13	14
1	0.00000005	0.00000005	0.00000005	0.00000005	0.00000006	0.00000096	0.00000003
2	0.00000333	0.00000333	0.00000334	0.00000334	0.00000124	0.00000658	0.00000004
3	0.00000334	0.00000334	0.00000335	0.00000335	0.00000125	0.00000659	0.00000004
4	0.00001072	0.00001072	0.00001073	0.00001073	0.00086173	0.00109214	0.00004186
5	0.00001071	0.00001071	0.00001072	0.00001072	0.00085812	0.00109100	0.00004174
6	0.00000124	0.00000651	0.00000125	0.00000652	0.00000159	0.00000350	0.00000106
7	0.00000658	0.00000004	0.00000659	0.00000004	0.00000349	0.00000113	0.00000099
8	0.00085979	0.00108737	0.00086340	0.00108851	0.00000879	0.00001982	0.00000702
9	0.00109157	0.00004180	0.00109271	0.00004192	0.00001976	0.00000710	0.00000671
10	0.00086340	0.00108851	0.00086700	0.00108966	0.00000880	0.00001986	0.00000703
11	0.00109271	0.00004192	0.00109385	0.00004204	0.00001980	0.00000710	0.00000672
12	0.00000879	0.00001987	0.00000880	0.00001991	0.00017405	0.00034520	0.00002917
13	0.00001982	0.00000702	0.00001986	0.00000703	0.00034580	0.00002910	0.00001944
14	0.00000710	0.00000671	0.00000710	0.00000672	0.00002904	0.00001965	0.00000056

GEODETIC COORDINATES OF P AND Q

I	LATITUDE(P)	LONGITUDE(P)	HEIGHT(P)	LATITUDE(Q)	LONGITUDE(Q)	HEIGHT(Q)
6	47.378100	115.174300	200420.0	44.000000	115.344700	10000.0

MATRIX OF COVARIANCES CALCULATED WITH COVAPP

	1	2	3	4	5	6	7
1	10.09432806	0.04255007	24.05859593	0.70313847	0.79308519	-4.50601941	0.15399810
2	0.03782843	0.00016018	0.09062344	0.00237339	0.00271192	-0.02235988	0.00376417
3	22.02717674	0.09332780	52.80399259	1.36242580	1.55965747	-13.42967126	0.45897358
4	0.62511358	0.00237339	1.32294679	0.01233951	0.01775454	-0.54593071	0.01865846
5	0.70507922	0.00271192	1.51446328	0.01775454	0.02334765	-0.59259387	0.02025254
6	4.37512023	0.02442015	14.24209103	0.59625535	0.64719627	0.59833482	0.06669019
7	-0.15885081	-0.00088664	-0.51709840	-0.02164870	-0.02349825	0.06031102	2.39794753
8	-0.69167251	-0.00423460	-2.49018917	-0.10612064	-0.11492193	0.05627435	-0.01569923
9	0.02511308	0.00015375	0.09041319	0.00385300	0.00417256	-0.01501233	-0.37890967
10	-0.63217480	-0.00390251	-2.29650928	-0.09801210	-0.10612064	0.06441117	-0.01479230
11	0.02295285	0.00014169	0.08338110	0.00355860	0.00385300	-0.01419215	-0.34629973
12	-0.12588855	0.00041769	0.30509396	0.05570736	0.05648248	0.34088185	0.00341174
13	-0.01490220	-0.00012386	-0.07446645	-0.00467645	-0.00493132	0.00271258	0.22478482
14	-0.53583132	-0.00298932	-1.74332015	-0.07292875	-0.07916465	0.20328904	-0.02265323

	8	9	10	11	12	13	14
1	0.77806347	-0.02659116	0.71113441	-0.02430379	-0.14153777	-0.01343315	-0.53409798
2	0.00423492	-0.00014473	0.00390280	-0.00013338	0.00041817	-0.00010495	-0.00264901
3	2.56469312	-0.08765117	2.36521852	-0.08083391	0.31449920	-0.06520105	-1.59095739
4	0.10612860	-0.00362706	0.09801945	-0.00334992	0.05572622	-0.00411815	-0.06462636
5	0.11493055	-0.00392788	0.10612860	-0.00362706	0.05650215	-0.00433379	-0.07015235
6	0.06146416	-0.01714706	0.07035138	-0.01615649	-0.37231294	-0.00278901	-0.23111806
7	-0.01639681	-0.41385397	-0.01550100	-0.37823664	-0.00197059	-0.20901664	0.02329147
8	-0.05607440	0.00452555	-0.05523854	0.00429236	0.05218872	0.00197097	0.05435964
9	0.00449226	0.07170837	0.00426928	0.06608032	0.00185358	0.05198288	-0.00562910
10	-0.05523854	0.00429236	-0.05427007	0.00407225	0.04712559	0.00193305	0.05121664
11	0.00426928	0.06608032	0.00405803	0.06093665	0.00182678	0.04914044	-0.00531236
12	-0.05219304	-0.00219759	-0.04712983	-0.00214691	-0.04559105	0.00058194	-0.01177057
13	-0.00209375	-0.05774589	-0.00205346	-0.05440711	0.00065966	-0.00935006	0.00398147
14	-0.05522107	0.00597976	-0.05220157	0.00564328	-0.00659349	0.00354050	0.07841849

MATRIX OF COVARIANCES CALCULATED WITH COVAXN

	1	2	3	4	5	6	7
1	10.09434248	0.04255033	24.05875701	0.70315793	0.79310518	-4.50631443	0.15400816
2	0.03782866	0.00016019	0.09062606	0.00237370	0.00271224	-0.02236462	0.00076434
3	22.02732421	0.09333049	52.80566738	1.36262150	1.55985845	-13.43270297	0.45907720
4	0.62513089	0.00237370	1.32313682	0.01257868	0.01762623	-0.54630934	0.01867071
5	0.70509700	0.00271224	1.51465844	0.01762623	0.02338924	-0.59296210	0.02026512
6	4.37540668	0.02442533	14.24530614	0.59664703	0.64759844	0.59839975	0.06669368
7	-0.15886121	-0.00088683	-0.51721513	-0.02166292	-0.02351285	0.06031403	2.39810456
8	-0.69181912	-0.00423730	-2.49186671	-0.10632065	-0.11512741	0.05626462	-0.01570181
9	0.02511840	0.00015385	0.09047409	0.00386026	0.00418002	-0.01501472	-0.37899001
10	-0.63231732	-0.00390514	-2.29814332	-0.09820678	-0.10632065	0.06440232	-0.01479484
11	0.02295603	0.00014179	0.08344043	0.00356567	0.00386026	-0.01419450	-0.34637794
12	-0.12590221	0.00041761	0.30505199	0.05568940	0.05646418	0.34051273	0.00342513
13	-0.01490298	-0.00012388	-0.07447924	-0.00467753	-0.00493244	0.00272675	0.22479609
14	-0.53586641	-0.00298995	-1.74371384	-0.07297673	-0.07921391	0.20329868	-0.02265437

	8	9	10	11	12	13	14
1	0.77822839	-0.02659680	0.71129494	-0.02430928	-0.14155313	-0.01343382	-0.53413295
2	0.00423762	-0.00014483	0.00390543	-0.00013347	0.00041810	-0.00010497	-0.00264957
3	2.56642084	-0.08771022	2.36690145	-0.08089142	0.31445604	-0.06521186	-1.59131668
4	0.10632863	-0.00363390	0.09821415	-0.00335658	0.05570808	-0.00411898	-0.06466887
5	0.11513694	-0.00393490	0.10632863	-0.00363390	0.05648385	-0.00433465	-0.07019600
6	0.06145553	-0.01714989	0.07034171	-0.01615927	-0.37190975	-0.00280358	-0.23112965
7	-0.01639943	-0.41394172	-0.01550357	-0.37832206	-0.00198602	-0.20902656	0.02329265
8	-0.05605617	0.00432659	-0.05522045	0.00429336	0.05188106	0.00198210	0.05436823
9	0.00449317	0.07175414	0.00427015	0.06612491	0.00186536	0.05199078	-0.00563000
10	-0.05522045	0.00429336	-0.05426387	0.00407361	0.04682342	0.00194397	0.05122507
11	0.00427015	0.06612491	0.00405931	0.06098007	0.00183835	0.04914821	-0.00531324
12	-0.05188541	-0.00220875	-0.04682769	-0.00215787	-0.04627026	0.00060000	-0.01181752
13	-0.00210557	-0.05775502	-0.00206507	-0.05441606	0.00067906	-0.00939980	0.00398668
14	-0.05522947	0.00598071	-0.05220982	0.00564422	-0.00664617	0.00354600	0.07842214

ABSOLUTE ERRORS

	1	2	3	4	5	6	7
1	0.00001441	0.00000026	0.00016108	0.00001947	0.00001999	0.00029502	0.00001008
2	0.00000023	0.00000000	0.00000262	0.00000031	0.00000031	0.00000474	0.00000016
3	0.00014748	0.00000269	0.00167479	0.00019570	0.00020098	0.00303171	0.00010361
4	0.00001731	0.00000031	0.00019003	0.00003917	0.00012831	0.00035863	0.00001226
5	0.00001777	0.00000031	0.00019516	0.00012831	0.00004159	0.00036824	0.00001258
6	0.00028645	0.00000518	0.00321511	0.00039168	0.00040217	0.00006493	0.00000349
7	0.00001040	0.00000019	0.00011673	0.00001422	0.00001460	0.00000301	0.00015703
8	0.00014661	0.00000270	0.00167754	0.00020001	0.00020548	0.00000973	0.00000259
9	0.00000532	0.00000010	0.00006091	0.00000726	0.00000746	0.00000240	0.00008034
10	0.00014271	0.00000263	0.00163404	0.00019469	0.00020001	0.00000885	0.00000234
11	0.00000518	0.00000010	0.00005933	0.00000707	0.00000726	0.00000235	0.00007821
12	0.00001366	0.00000007	0.00004197	0.00001815	0.00001830	0.00036912	0.00001339
13	0.00000078	0.00000002	0.00001279	0.00000108	0.00000113	0.00001417	0.00001127
14	0.00003509	0.00000063	0.00039370	0.00004798	0.00004926	0.00000965	0.00000114

	8	9	10	11	12	13	14
1	0.00016492	0.00000364	0.00016054	0.00000549	0.00001536	0.00000067	0.00003498
2	0.00000270	0.00000009	0.00000263	0.00000009	0.00000007	0.00000002	0.00000056
3	0.00172773	0.00005905	0.00168293	0.00005752	0.00004317	0.00001082	0.00035929
4	0.00020003	0.00000684	0.00019470	0.00000665	0.00001815	0.00000083	0.00004252
5	0.00020550	0.00000702	0.00020003	0.00000684	0.00001839	0.00000087	0.00004365
6	0.00001063	0.00000283	0.00000967	0.00000277	0.00040319	0.00001457	0.00001159
7	0.00000262	0.00008773	0.00000257	0.00008542	0.00001543	0.00000992	0.00000117
8	0.00001823	0.00000104	0.00001898	0.00000100	0.00030765	0.00001112	0.00000859
9	0.00000090	0.00004578	0.00000087	0.00004458	0.00001178	0.00000790	0.00000090
10	0.00001808	0.00000100	0.00000620	0.00000137	0.00030217	0.00001093	0.00000843
11	0.00000087	0.00004458	0.00000126	0.00004341	0.00001157	0.00000777	0.00000088
12	0.00030763	0.00001116	0.00030215	0.00001096	0.00067921	0.00001806	0.00004695
13	0.00001182	0.00000913	0.00001161	0.00000896	0.00001940	0.00004974	0.00000522
14	0.00000839	0.00000096	0.00000825	0.00000094	0.00005268	0.00000351	0.00000363

	8	9	10	11	12	13	14
1	0.77822839	-0.02659680	0.71129494	-0.02430928	-0.14155313	-0.01343382	-0.53413295
2	0.00423762	-0.00014483	0.00390543	-0.00013347	0.00041810	-0.00010497	-0.00264957
3	2.56642084	-0.08771022	2.36690145	-0.08089142	0.31445604	-0.06521186	-1.59181668
4	0.10632863	-0.00363390	0.09821415	-0.00335658	0.05570808	-0.00411898	-0.06466887
5	0.11513604	-0.00393490	0.10632863	-0.00363390	0.05648385	-0.00433465	-0.07019600
6	0.06145353	-0.01714989	0.07034171	-0.01615927	-0.37190975	-0.00280358	-0.23112965
7	-0.01639943	-0.41394172	-0.01550357	-0.37832206	-0.00198602	-0.20902656	0.02329265
8	-0.05605617	0.00452659	-0.05522045	0.00429336	0.05188106	0.00198210	0.05436823
9	0.00449317	0.07175414	0.00427015	0.06612491	0.00186536	0.05199078	-0.00563000
10	-0.05522045	0.00429336	-0.05426387	0.00407361	0.04682342	0.00194397	0.05122507
11	0.00427015	0.06612491	0.00405931	0.06098007	0.00183835	0.04914821	-0.00531324
12	-0.05188541	-0.00220875	-0.04682769	-0.00215787	-0.04627026	0.00060000	-0.01181752
13	-0.00210557	-0.05775502	-0.00206507	-0.05441606	0.00067906	-0.00939980	0.00398668
14	-0.05522947	0.00598071	-0.05220982	0.00564422	-0.00664617	0.00354600	0.07842214

RELATIVE ERRORS

	1	2	3	4	5	6	7
1	0.00000143	0.00000610	0.00000670	0.00002768	0.00002521	0.00006547	0.00006547
2	0.00000610	0.00002629	0.00002887	0.00012885	0.00011591	0.00021192	0.00021192
3	0.00000670	0.00002887	0.00003172	0.00014362	0.00012885	0.00022570	0.00022570
4	0.00002768	0.00012885	0.00014362	0.00311429	0.00727946	0.00065646	0.00065646
5	0.00002521	0.00011591	0.00012885	0.00727946	0.00177827	0.00062101	0.00062101
6	0.00006547	0.00021192	0.00022570	0.00065646	0.00062101	0.00010851	0.00005227
7	0.00006547	0.00021192	0.00022570	0.00065646	0.00062101	0.00004996	0.00006548
8	0.00021192	0.00063704	0.00067321	0.00188120	0.00178480	0.00017299	0.00016478
9	0.00021192	0.00063704	0.00067321	0.00188120	0.00178480	0.00015955	0.00021199
10	0.00022570	0.00067321	0.00071103	0.00198241	0.00188120	0.00013742	0.00017168
11	0.00022570	0.00067321	0.00071103	0.00198241	0.00188120	0.00016588	0.00022578
12	0.00010849	0.00017318	0.00013757	0.00032594	0.00032415	0.00108401	0.00390801
13	0.00005227	0.00016478	0.00017168	0.00023180	0.00022817	0.00519800	0.00005014
14	0.00006548	0.00021199	0.00022578	0.00065743	0.00062190	0.00004746	0.00005042

	8	9	10	11	12	13	14
1	0.00021192	0.00021192	0.00022570	0.00022570	0.00010852	0.00004996	0.00006548
2	0.00063704	0.00063704	0.00067321	0.00067321	0.00017279	0.00015955	0.00021199
3	0.00067321	0.00067321	0.00071103	0.00071103	0.00013727	0.00016588	0.00022578
4	0.00188120	0.00188120	0.00198241	0.00198241	0.00032576	0.00020246	0.00065745
5	0.00178480	0.00178480	0.00188120	0.00188120	0.00032396	0.00020018	0.00062190
6	0.00017299	0.00016478	0.00013742	0.00017168	0.00108412	0.00519800	0.00005014
7	0.00015955	0.00021199	0.00016588	0.00022578	0.00776994	0.00004746	0.00005042
8	0.00032518	0.00022980	0.00032748	0.00023334	0.00592998	0.00561187	0.00015799
9	0.00020119	0.00063798	0.00020336	0.00067424	0.00631412	0.00015200	0.00015987
10	0.00032748	0.00023334	0.00011421	0.00033534	0.00645341	0.00561999	0.00016461
11	0.00020336	0.00067424	0.00031050	0.00071194	0.00629274	0.00015804	0.00016639
12	0.00592901	0.00505229	0.00645231	0.00507926	0.01467926	0.03009641	0.00397333
13	0.00561187	0.00015799	0.00561999	0.00016461	0.02856868	0.00529127	0.00130812
14	0.00015200	0.00015987	0.00015804	0.00016639	0.00792696	0.00155270	0.00004655

GEODETIC COORDINATES OF P AND Q

I	LATITUDE(P)	LONGITUDE(P)	HEIGHT(P)	LATITUDE(Q)	LONGITUDE(Q)	HEIGHT(Q)
8	-40.000000	217.344700	200370.0	-47.378100	215.174300	283200.0

MATRIX OF COVARIANCES CALCULATED WITH COVAPP

	1	2	3	4	5	6	7
1	0.65539283	0.00094300	0.45043070	-0.01432226	-0.01217016	-1.66842312	-0.37811352
2	0.00099146	-0.00000280	-0.00213021	-0.00018994	-0.00019514	-0.00584126	-0.00132380
3	0.46767889	-0.00210369	-1.52619170	-0.12080173	-0.12481933	-3.37050252	-0.76385453
4	-0.01505814	-0.00018994	-0.12232440	-0.00621381	-0.00660226	-0.08645893	-0.01959412
5	-0.01279547	-0.00019514	-0.12639263	-0.00660226	-0.00699513	-0.09885364	-0.02240313
6	1.69891091	0.00565733	3.30551607	0.08373645	0.09574087	-0.43455747	-0.18043850
7	0.33845249	0.00112704	0.65851608	0.01668175	0.01907324	-0.17876815	0.37087109
8	-0.18949676	-0.00061254	-0.35640250	-0.00804538	-0.00934740	0.06894484	0.02476620
9	-0.03775104	-0.00012203	-0.07100155	-0.00160278	-0.00186216	0.02401865	-0.04044263
10	-0.16639257	-0.00053561	-0.31144944	-0.00690662	-0.00804538	0.06303511	0.02231234
11	-0.03314828	-0.00010670	-0.06204612	-0.00137592	-0.00160278	0.02158751	-0.03539900
12	0.09635993	0.00045571	0.27720630	0.01002323	0.01097377	0.04675141	-0.00728467
13	0.03498853	0.00014337	0.08593021	0.00277516	0.00307611	-0.00855862	0.03880753
14	-0.07191500	-0.00023412	-0.13636184	-0.00333526	-0.00383271	0.03795807	0.02601471

	8	9	10	11	12	13	14
1	0.17923285	0.04061941	0.15738007	0.03566693	0.08969381	0.04219449	-0.08753638
2	0.00060913	0.00013805	0.00053262	0.00012071	0.00044736	0.00017794	-0.00029962
3	0.35000668	0.07932176	0.30386032	0.06931690	0.26881825	0.10509769	-0.17233828
4	0.00800059	0.00181317	0.00686817	0.00155653	0.00986326	0.00336848	-0.00426834
5	0.00929537	0.00210660	0.00800059	0.00181317	0.01079627	0.00374238	-0.00490495
6	0.06640213	0.02385281	0.06071035	0.02148945	-0.04525344	0.00932499	-0.04228250
7	0.02313283	-0.03895109	0.02079125	-0.03409347	0.01024937	-0.04135697	-0.02834416
8	-0.01043309	-0.00331770	-0.00953006	-0.00299332	0.00297172	-0.00198757	0.00564574
9	-0.00315084	0.00407095	-0.00283625	0.00354124	-0.00202625	0.00540682	0.00381618
10	-0.00953006	-0.00299332	-0.00869365	-0.00269863	0.00235639	-0.00186076	0.00507073
11	-0.00283625	0.00354124	-0.00255135	0.00307808	-0.00188686	0.00484439	0.00343072
12	-0.00292327	0.00167795	-0.00231093	0.00158253	-0.00796677	-0.00040416	-0.00158678
13	0.00175695	-0.00499065	0.00164483	-0.00448235	-0.00025942	-0.00185327	-0.00255593
14	-0.00477944	-0.00337337	-0.00428227	-0.00303264	-0.00201882	-0.00288990	0.00566117

MATRIX OF COVARIANCES CALCULATED WITH COVAXN

	1	2	3	4	5	6	7
1	0.65546125	0.00094347	0.45072312	-0.01430718	-0.01215412	-1.66848484	-0.37812751
2	0.00099195	-0.00000280	-0.00212813	-0.00018984	-0.00019503	-0.00584172	-0.00132390
3	0.46798251	-0.00210164	-1.52490499	-0.12073647	-0.12474974	-3.37078590	-0.76391876
4	-0.01504228	-0.00018984	-0.12225831	-0.00621153	-0.00659731	-0.0864747l	-0.01959770
5	-0.01277860	-0.00019503	-0.12632217	-0.00659731	-0.00699255	-0.09887036	-0.02240692
6	1.69897376	0.00565777	3.30579399	0.08375173	0.09575706	-0.43461876	-0.18045542
7	0.33846501	0.00112713	0.65857145	0.01668480	0.01907647	-0.17878377	0.37088277
8	-0.18951161	-0.00061265	-0.35646975	-0.00804899	-0.00935123	0.06896661	0.02477185
9	-0.03775400	-0.00012205	-0.07101493	-0.00160350	-0.00186293	0.02402379	-0.04044506
10	-0.16640656	-0.00053571	-0.31151289	-0.00691001	-0.00804899	0.06305605	0.02231776
11	-0.03315107	-0.00010672	-0.06205877	-0.00137659	-0.00160350	0.02159244	-0.03540127
12	0.09637347	0.00045585	0.27729814	0.01003113	0.01098196	0.04674616	-0.00728748
13	0.03499181	0.00014341	0.08397109	0.00277688	0.00307789	-0.00856129	0.03881076
14	-0.07191726	-0.00023414	-0.13637039	-0.00333563	-0.00383311	0.03796099	0.02601698

	8	9	10	11	12	13	14
1	0.17924689	0.04062259	0.15739330	0.03566993	0.08970651	0.04219818	-0.08753913
2	0.00060924	0.00013807	0.00053272	0.00012073	0.00044750	0.00017798	-0.00029964
3	0.35007273	0.07933673	0.30592264	0.06933103	0.26890778	0.10512170	-0.17234934
4	0.00800418	0.00181398	0.00687154	0.00155729	0.00987106	0.00337045	-0.00426881
5	0.00929917	0.00210746	0.00800418	0.00181398	0.01080436	0.00374443	-0.00490546
6	0.06642310	0.02385825	0.06073052	0.02149467	-0.04524845	0.00932790	-0.04228601
7	0.02313778	-0.03895343	0.02079610	-0.03409566	0.01025211	-0.04136016	-0.02834663
8	-0.01044109	-0.00331968	-0.00953777	-0.00299522	0.00296982	-0.00198859	0.00564690
9	-0.00315262	0.00407139	-0.00283796	0.00354164	-0.00202721	0.00540785	0.00381700
10	-0.00953777	-0.00299522	-0.00871187	-0.00270291	0.00235447	-0.00186174	0.00507184
11	-0.00283796	0.00354164	-0.00255514	0.00307796	-0.00188779	0.00484538	0.00343150
12	-0.00292135	0.00167890	-0.00230907	0.00158345	-0.00813464	-0.00044175	-0.00159595
13	0.00175785	-0.00499167	0.00164571	-0.00448333	-0.00029244	-0.00186136	-0.00255818
14	-0.00478036	-0.00337410	-0.00428315	-0.00303333	-0.00202596	-0.00289192	0.00566121

ABSOLUTE ERRORS

	1	2	3	4	5	6	7
1	0.00006842	0.00000047	0.00029243	0.00001508	0.00001605	0.00006172	0.00001399
2	0.00000049	0.00000000	0.00000208	0.00000011	0.00000011	0.00000046	0.00000010
3	0.00030362	0.00000205	0.00128671	0.00006526	0.00006958	0.00028338	0.00006422
4	0.00001586	0.00000011	0.00006609	0.00000228	0.00000494	0.00001578	0.00000358
5	0.00001687	0.00000011	0.00007046	0.00000494	0.00000258	0.00001672	0.00000379
6	0.00006285	0.00000044	0.00027792	0.00001528	0.00001619	0.00006129	0.00001692
7	0.00001252	0.00000009	0.00005537	0.00000304	0.00000323	0.00001562	0.00001168
8	0.00001484	0.00000011	0.00006725	0.00000360	0.00000382	0.00002177	0.00000565
9	0.00000296	0.00000002	0.00001340	0.00000072	0.00000076	0.00000514	0.00000243
10	0.00001399	0.00000010	0.00006345	0.00000339	0.00000360	0.00002094	0.00000542
11	0.00000279	0.00000002	0.00001264	0.00000068	0.00000072	0.00000493	0.00000227
12	0.00001354	0.00000014	0.00009184	0.00000790	0.00000819	0.00000526	0.00000281
13	0.00000328	0.00000003	0.00002088	0.00000172	0.00000178	0.00000267	0.00000322
14	0.00000226	0.00000001	0.00000875	0.00000037	0.00000040	0.00000293	0.00000227

	8	9	10	11	12	13	14
1	0.00001404	0.00000318	0.00001323	0.00000300	0.00001270	0.00000369	0.00000276
2	0.00000011	0.00000062	0.00000010	0.00000002	0.00000014	0.00000004	0.00000002
3	0.00006605	0.00001497	0.00006231	0.00001412	0.00008953	0.00002400	0.00001105
4	0.00000358	0.00000081	0.00000338	0.00000077	0.00000780	0.00000197	0.00000047
5	0.00000380	0.00000086	0.00000358	0.00000081	0.00000809	0.00000205	0.00000051
6	0.00002097	0.00000544	0.00002017	0.00000522	0.00000499	0.00000291	0.00000351
7	0.00000495	0.00000234	0.00000475	0.00000219	0.00000274	0.00000319	0.00000247
8	0.00000800	0.00000198	0.00000771	0.00000190	0.00000190	0.00000102	0.00000116
9	0.00000178	0.00000043	0.00000171	0.00000040	0.00000096	0.00000104	0.00000082
10	0.00000771	0.00000190	0.00001823	0.00000428	0.00000183	0.00000098	0.00000111
11	0.00000171	0.00000040	0.00000380	0.00000012	0.00000092	0.00000099	0.00000078
12	0.00000192	0.00000095	0.00000185	0.00000092	0.00016787	0.00003760	0.00000917
13	0.00000090	0.00000103	0.00000087	0.00000098	0.00003302	0.00000809	0.00000224
14	0.00000092	0.00000072	0.00000088	0.00000069	0.00000714	0.00000203	0.00000005

	8	9	10	11	12	13	14
1	0.17924689	0.04062259	0.15739330	0.03566993	0.08970651	0.04219818	-0.08753913
2	0.00060924	0.00013807	0.00053272	0.00012073	0.00044750	0.00017798	-0.00029964
3	0.35007273	0.07933673	0.30592264	0.06933103	0.26890778	0.10512170	-0.17234934
4	0.00800418	0.00181398	0.00687154	0.00155729	0.00987106	0.00337045	-0.00426881
5	0.00929917	0.00210746	0.00800418	0.00181398	0.01080436	0.00374443	-0.00490546
6	0.06642310	0.02385825	0.06073052	0.02149467	-0.04524845	0.00932790	-0.04228601
7	0.02313778	-0.03895343	0.02079610	-0.03409566	0.01025211	-0.04136016	-0.02834663
8	-0.01044109	-0.00331968	-0.00953777	-0.00299522	0.00296982	-0.00198859	0.00564690
9	-0.00315262	0.00407139	-0.00283796	0.00354164	-0.00202721	0.00540785	0.00381700
10	-0.00953777	-0.00299522	-0.00871187	-0.00270291	0.00285447	-0.00186174	0.00507184
11	-0.00283796	0.00354164	-0.00255514	0.00307796	-0.00188779	0.00484538	0.00343150
12	-0.00292135	0.00167890	-0.00230907	0.00158345	-0.00813464	-0.00044175	-0.00159595
13	0.00175785	-0.00499167	0.00164571	-0.00448333	-0.00029244	-0.00186136	-0.00255818
14	-0.00478036	-0.00337410	-0.00428315	-0.00303333	-0.00202596	-0.00289192	0.00566121

RELATIVE ERRORS

	1	2	3	4	5	6	7
1	0.00010438	0.00049523	0.00064879	0.00105432	0.00132026	0.00003699	0.00003699
2	0.00049523	0.00118948	0.00097733	0.00055778	0.00057806	0.00007833	0.00007833
3	0.00064879	0.00097733	0.00084379	0.00054055	0.00055778	0.00008407	0.00008407
4	0.00105432	0.00055778	0.00054055	0.00036641	0.00074905	0.00018246	0.00018246
5	0.00132026	0.00057806	0.00055778	0.00074905	0.00036847	0.00016911	0.00016911
6	0.00003699	0.00007833	0.00008407	0.00018246	0.00016911	0.00014102	0.00009377
7	0.00003699	0.00007833	0.00008407	0.00018246	0.00016911	0.00008737	0.00003149
8	0.00007833	0.00017481	0.00018867	0.00044771	0.00040891	0.00031569	0.00022809
9	0.00007833	0.00017481	0.00018867	0.00044771	0.00040891	0.00021408	0.00006006
10	0.00008407	0.00018867	0.00020370	0.00049129	0.00044771	0.00033206	0.00024286
11	0.00008407	0.00018867	0.00020370	0.00049129	0.00044771	0.00022835	0.00006413
12	0.00014047	0.00031482	0.00033119	0.00078715	0.00074564	0.00011245	0.00038627
13	0.00009377	0.00022809	0.00024286	0.00061763	0.00057891	0.00031185	0.00008300
14	0.00003149	0.00006006	0.00006413	0.00011029	0.00010355	0.00007710	0.00008724

	8	9	10	11	12	13	14
1	0.00007833	0.00007833	0.00008407	0.00008407	0.00014156	0.00008737	0.00003149
2	0.00017481	0.00017481	0.00018867	0.00018867	0.00031656	0.00021408	0.00006006
3	0.00018867	0.00018867	0.00020370	0.00020370	0.00033294	0.00022835	0.00006413
4	0.00044771	0.00044771	0.00049129	0.00049129	0.00079013	0.00058579	0.00011029
5	0.00040891	0.00040891	0.00044771	0.00044771	0.00074861	0.00054806	0.00010355
6	0.00031569	0.00022809	0.00033206	0.00024286	0.00011031	0.00031185	0.00008300
7	0.00021408	0.00006006	0.00022835	0.00006413	0.00026763	0.00007710	0.00008724
8	0.00076578	0.00059605	0.00080841	0.00063591	0.00063878	0.00051283	0.00020567
9	0.00056472	0.00010638	0.00060360	0.00011331	0.00047278	0.00019178	0.00021420
10	0.00080841	0.00063591	0.00209223	0.00158319	0.00077689	0.00052652	0.00021958
11	0.00060360	0.00011331	0.00148646	0.00003764	0.00048794	0.00020509	0.00022847
12	0.00065793	0.00056853	0.00080257	0.00057951	0.02063669	0.08511429	0.00374338
13	0.00051283	0.00020567	0.00052652	0.00021958	0.11291470	0.00434540	0.00087656
14	0.00019178	0.00021420	0.00020509	0.00022847	0.00352235	0.00070066	0.00000826